Factions and Political Competition∗

Nicola Persico†  José C. R. Pueblita
New York University and NBER  Ministry of Finance, Mexico

Dan Silverman‡
University of Michigan and NBER

March 22, 2011

Abstract

This paper presents a new model of political competition where candidates belong to factions. Before elections, factions compete to direct local public goods to their local

∗We thank Lucas Davis, Allan Drazen, Robert Inman, Michael Laver, Alessandro Lizzeri, Eric Maskin, Ennio Stacchetti, Luis Videgaray, and participants in a number of seminars for their helpful comments. Special thanks go to the Editor (Canice Prendergast) and to an anonymous referee for their many suggestions that importantly improved the paper.

†Department of Economics and School of Law, 19 W. 4th Street, New York, NY 10012. nicola.persico@nyu.edu.

‡Department of Economics, 611 Tappan Street, Ann Arbor, MI 48109-1220. dansilv@umich.edu.
constituencies. The model of factional competition delivers a rich set of implications relating the internal organization of the party to the allocation of resources. In doing so, the model provides a unified explanation of two prominent features of public resource allocations: the persistence of (possibly inefficient) policies, and the tendency of public spending to favor incumbent party strongholds over swing constituencies.
1 Introduction

This paper presents a new model of political competition where candidates belong to intra-party factions. Before elections, hierarchical networks of party officials (factions) work to direct local public goods to their constituencies and thereby win votes and advance their careers within the party. The model delivers a rich set of implications linking the allocation of public resources to the internal organization of the party. In doing so, the model provides a new and unified explanation of two prominent features of public resource allocations: the persistence of (possibly inefficient) policies, and the tendency of public spending to favor incumbent party strongholds over swing constituencies. Importantly, the model predicts policy persistence and stronghold spending even if there is turnover in the individuals that hold (senior) positions in the faction.

A vast formal literature has investigated the connection between elections and the allocation of public spending.¹ Virtually all of this literature treats competing political agents as singletons, be they candidates or parties, vested with the power to deliver or promise resources. This view is often oversimplified. In reality, the power to deliver public resources to a constituency is often dispersed among (networks of) party and government officials. To illustrate, consider the well-documented case of Lyndon Johnson and his successful efforts

---

¹This literature includes models where candidates can commit to policies—in one dimension (median voter, see Black 1948) or many dimensions (distributive politics, see Lindbeck and Weibull 1987)—and models where candidates cannot commit (citizen candidates, see Osborne and Slivinski 1996, Besley and Coate 1997). There are also agency models (Barro 1973, Persson, Roland and Tabellini 1997) and signaling models (the political cycle, Rogoff 1990), just to name a few.
as a first-term U.S. Congressman to bring a massive New Deal dam project to his district in Texas.\(^2\) Johnson needed to secure land rights, mobilize local support, obtain Congressional and regulatory approvals, and ensure both the appropriation of funds and their timely disbursement. Each of these processes was complex and fraught with political and legal obstacles. To achieve all this, Johnson tapped a network of contacts in the Democratic party to help with each step. This network ranged from the party rank and file in Texas, to Congressional leaders, to White House officials, each with an incentive to assist Johnson and his constituents. By this account, and others like it (see Section 2, below), the political allocation of resources results from a team effort: it depends on the size and power of the party faction available to each local representative.

This paper formalizes the notion that power is dispersed across a party hierarchy. We model the distribution of power across networks of party members (fractions) and study the effects of this power pattern on the allocation of public resources. We base our investigation on an exceedingly simple model, treating the faction as a team of fellow party officers. In the model, each of many districts holds an election in which a party officer competes against a challenger. To win, it helps the office-holder to deliver local public goods before the election; and this delivery requires the assistance of fellow party officers. If it is in their self-interest, these fellow officers can work to help bring public resources to a district. (This team of party officers corresponds to the network that helped Lyndon Johnson with the dam.) The party’s promotion policy incentivizes faction members to lend a hand at election time; this is because factions are aggregates, or networks, of politicians who share the same career fate:

\(^2\)A vivid account is provided in Caro (1983, pp. 370-385 and 459-468).
when intra-party reshufflings occur and posts are assigned, either (all) faction members are promoted or they are (all) passed over. At election time, then, all faction members have an interest in working to direct pork to the constituents of their faction’s candidate. The size of a faction, and hence its power, evolves over time: a faction expands only if it wins elections—otherwise, it becomes marginalized within the party. Larger factions are better able to deliver pork.

Our basic model is simple, but delivers a rich set of novel implications for resource allocation. First, persistence. Over time, a faction that survives becomes more powerful and more able to deliver pork. As this happens, voters become less likely to vote it (and the party) out of office. Thus the model offers a novel, joint explanation for persistence of policies and incumbency advantage. Leading models of policy persistence emphasize forces outside of parties—either vested interests facing switching costs (Coate and Morris 1999), or voters who are uncertain of the gains from reform (Fernandez and Rodrik 1991).3 The factional model identifies an additional source of persistence—the persistence of factions within the party hierarchy. Powerful factions take time to build, but once built, they are resilient—they become durable reservoirs of power for special interests (geographic or otherwise).

A second implication of the model is a stronghold premium. In the standard, static models of distributive politics (Lindbeck and Weibull 1987, for example), a monolithic party allocates a given public budget across localities to maximize the sum of the probabilities of winning. In these models, “swing” districts are the focus of pork spending as their votes are the most responsive to public largesse; localities (or groups) that are loyal to the party, or “party

3See, also, Hassler et al (2003), Majumdar and Mukand (2004), and Mitchell and Moro (2006).
“strongholds,” are predicted to receive relatively little. Tests of the standard models have produced mixed results. A number of studies, of many different countries, either find little evidence that spending is directed to swing constituencies or that ruling party strongholds benefit disproportionately from public expenditures. The factional model accounts for this “stronghold premium.” In the model, the premium arises because party strongholds tend to elect the party’s candidate, and so over time their factions become powerful and thus more successful at procuring public resources.

Importantly, the model predicts both policy persistence and a stronghold premium even if the individuals that compose factions or hold offices turn over. Thus, the model offers an explanation for an incumbency advantage in the absence of either seniority rules for legislators or selection of incumbents based on political talent.

Finally, the model links public spending to political careers; patterns of party promotions are predictive of the allocation of funds. The exact nature of these predictions will depend somewhat on the details of each party’s internal rules.

---

4 This literature is discussed in Section 1.1. Motivated in part by evidence of stronghold spending, Cox and McCubbins (1986) offer a prominent alternative to the “swing-voter” models, in which incumbent strongholds or “core-voters” are favored by pork spending because they are more responsive than opposition voters and not as risky as swing voters.

5 We do not contend that factions are the only source of the “stronghold premium.” There may be other features of party organization that confer special advantage to strongholds.

6 Such is the case, for example, in Mexico where by law office-holders cannot be re-elected and yet districts blessed with powerful factions enjoy durable largesse (Camp, 2003). In another example, Lyndon Johnson’s faction, discussed above, was also persistent despite turnover. Johnson largely inherited it from James P. Buchanan, the 12-term Congressman whose death in 1937 left open the seat that Johnson then won.
The main contribution of the paper is to present, for the first time, a simple model of how factions influence public spending. In Section 6, we extend this simple model and thereby evaluate the robustness of its predictions. The extensions also allow us to investigate which features of factions, and the political systems in which they operate, drive their effects on the allocation of public resources. This section is organized, in part, around the question of why intra-party factions play prominent roles in many settings (Mexico, Italy, Japan, Chicago) but not at the national level of U.S. politics.

The analysis in this paper isolates the instrumental incentives (career concerns) for faction formation and the maintenance of factional loyalties. In reality, ideology and personal affinities are undoubtedly important in forming and sustaining factional links. Our hope is that future research will investigate the effects of these forces, and their interaction with instrumental motives for faction formation, on political competition and the allocation of public resources.

1.1 Related literature

There is a large descriptive literature in political science on party factions. Much of that research addresses themes that are central to our model—the effect of factions on the allocation of public spending, and the exchanges that sustain factional links. General theories of party factions are discussed in Belloni and Beller (1978) and Kato and Mershon (2006). See our section 2 below for more from the political science literature. More recent (and formal) papers are Eguia (2011) and Mutlu (2010 a,b)
Our paper also relates to a literature in economics on collusion in hierarchies. See e.g. Tirole (1986), or Carrillo (2000). Strictly speaking, ours is not a model of corruption or even collusion; indeed, we show that factional links have benefits for the party because they motivate effort by officials who would otherwise be unaffected by local election outcomes. Nevertheless, our paper can be seen as a first effort to apply some themes from that literature to political parties. Dal Bó et al. (2009) on familial legacies in the U.S. Congress is a related paper with an empirical focus.

Our model opens the black box of internal party organization. In so doing, it also relates to small literatures on platform competition in a non-unitary party, and on the effect of party charters on platforms. See Roemer (1999), Caillaud and Tirole (2002), Testa (2003), Castanheira et al. (2010).

1.1.1 Empirics on Distributive Politics

Finally, our paper relates to an empirical literature, dating at least to Wright (1974), that evaluates standard models of distributive politics. Larcinese, et al. (2006), and Larcinese et al. (2008) are recent examples and provide reviews.

The results in this literature are mixed. Important for our paper, studies in this literature often finds little evidence that public spending favors swing districts. In many contexts, indeed, there is evidence that spending favors party strongholds. In the U.S. context, for
example, Ansolabehere and Snyder (2006) find that counties with the highest vote shares for the governing party of a state receive the most state transfers. Stronghold spending has been found in a number of non-U.S. contexts as well. Examples include Joanis (2008) on Canada, Arulampalam et al. (2008) on India, Leigh (2008) on Australia, and Estevez, et al. (2002) on Mexico. Table 1 summarizes some of the more recent studies that either find no evidence that public spending favors swing districts, or instead that find evidence that spending favors party strongholds.

1.2 Plan of the paper

The paper proceeds as follows. In Section 2 we describe several examples of factions from a variety of political systems and identify key features they share. In Section 3 we set up the model. Section 4 shows that our model nests the familiar model of distributive politics as a special case, the case where the power to deliver public expenditure is not distributed across the party hierarchy. In Section 5 we study the resource allocation in a factional equilibrium. In Section 6 we further discuss the model and extend it in order to evaluate its robustness. Section 7 concludes.

2 Facts About Factions

In this section we briefly discuss factions as they arise in several political systems. The goal is to familiarize the reader with the phenomenon, and to show that factions share common
traits. Specifically, in each political system we will highlight, first, the hierarchical nature of relationships inside a faction, second, the nature of the exchange between patrons and clients, and third, the effects of factions on public expenditures.

2.1 What are Factions?

We begin with a broad definition taken from Zuckerman’s (1975) study of Italian factions.

I define a political party faction as a structured group within a political party which seeks, at a minimum, to control authoritative decision-making positions of the party. It is a “structured group” in that there are established patterns of behavior and interaction for the faction members over time. Thus, party factions are to be distinguished from groups that coalesce around a specific or temporarily limited issue and then dissolve [...] (Zuckerman, 1975, p. 20).

This definition highlights the durability of factions and refers to what we will call “factions of interest.” Zuckerman distinguishes these groups from “factions of principle,” i.e., lobbies organized around particular policy agendas. Factions of interest are less idealistic aggregations that pursue their own power rather than more general-interest policies. Bettcher (2005 pp. 343-4) further defines factions of interest, though he calls them clienteles.

Clienteles have a pyramidal structure built up from patron–client relationships.

In a political party, clienteles organize vertical relations among elected politicians

7 The terms “factions of interest” and “factions of principle” are borrowed from Bettcher (2005). Factions of principle appear prominently in U.K. and U.S. parties, for example.
and party officers, and these relations may extend outward and downward into different levels of government and party organization. The relationships – and thus the overall structure – are maintained through exchanges among individuals at different levels. Lower members (clients) deliver votes to their superiors (patrons), and in exchange receive selective incentives such as money, jobs, and services. ... [M]embers join and remain in the clientele for particularistic, self-interested reasons. Continued membership in the clientele also depends on an ongoing relationship with a particular patron. Consequently, clienteles are not firmly organized and become vulnerable to collapse if key patrons are lost.

Our paper is concerned with factions of interest.

2.2 Factions in Italy’s DC

The Christian Democratic Party (DC) dominated Italian government from the post-war period until the mid 1990s and its factions, called correnti, were quite formal organizations. Bettcher (2005, p. 351) reports:

Each faction acquired a common identity and common resources. The factions possessed well-developed organizational features, including: ‘formalized faction names, more or less distinct memberships, leadership cadres and chains of command, faction headquarters, communications networks including press organs, and faction finances’ (Belloni, 1978: 93). As of 1986, the factions all had offices clustered in historic Rome (Panorama, 15 June 1986: 49–50). Meetings
and conventions were held regularly at various levels at least through the 1980s (L’Espresso, 19 February 1989: 8).

Faction members are described by Zuckerman (1975, p. 40) as following three rules:

(1) Seek to control cabinet positions. Strive to occupy more and “better” positions than previously held and to defend those already controlled.

(2) Seek to further the career of the leader. Support him in his effort to achieve “better” positions.

(3) Seek to obtain goods of value to those who are not faction members only when the persistence of the faction or the strength of the Christian Democrat Party is at stake.

DC factions were typical in that they were not organized around ideology or broad-based policy goals. One longtime factional leader and cabinet minister contended:

“The number of factions has now grown to nine. This is due to personal power games within the party. When a new faction forms, such as the Tavianei, or the Morotei, it must justify itself in ideological terms, but this is artificial. The factions are power groups.” (Quoted in Zuckerman, 1975, p. 26.)

While not primarily motivated by policy, DC factions had a substantial impact on the distribution of public resources. Bettcher (2005, pp. 351-2) reports that
Christian Democratic factions competed vigorously on behalf of their members for seats in the cabinet and the party’s National Council. [...] The factions also procured and distributed a much broader range of patronage, including public jobs at all levels. They colonized the state thoroughly and diverted its resources for their purposes [...]. The Italian regime was infamous for partitocrazia, a system in which political parties held preponderance over all aspects of government and society. The DC received the lion’s share of ministries, especially the most coveted ones (for example, Agriculture, Post and Telecommunications, and State Holdings) (Leonardi and Wertman, 1989: 225–36). [...] At the local level, from Palermo and Naples to Genoa and the Veneto, DC factions divided up and governed hospitals, welfare agencies, public utilities, credit agencies, housing and construction agencies, chambers of commerce, cooperatives, industrial associations, and professional associations (Caciagli, 1977: Ch. 6; Tamburrano, 1974: 111–16). Public entities proliferated to meet the expanding needs of the DC and its factions.

2.3 Factions in Japan’s LDP

The Liberal Democratic Party (LDP) led the Japanese national government almost continuously from the party’s formation in 1955 until its defeat in 2009. The great majority of LDP politicians have been long-term members of factions. These factions were called shidan (divisions) or gundan (army corps). Like Italian factions, they were formal, hierarchical
Offices proliferated within the largest factions as they matured. These offices had regular functions and procedures, which became standardized across the different factions (Ishikawa and Hirose, 1989: 212). The first of these was the faction secretary-general (jimu socho), analogous to the secretary-general of the party. [...] The secretary-general of each faction was entrusted with the daily business of his faction, including keeping order in the faction and handling relations with other factions. [...] Next was the standing secretariat (jonin kanjikai), which determined a faction’s management policies. It met prior to weekly faction meetings and then obtained approval of its decisions from the full faction (Iseri, 1988: 30–2, 34–5; Ishikawa and Hirose, 1989: 213). Under the standing secretariat were one or more bureaus (kyoku), charged with executing its internal policies. Some factions had specialized bureaus for handling policy issues or elections. The secretariats and the bureaus were specialized, permanent, hierarchical structures within the faction, governed by a set of written faction rules. They curtailed the influence of the leader and diminished the impact of his individual characteristics on the faction (Iseri, 1988: 32–5).

As in Italy, Japanese factions were based on mutual dependence between patrons and clients. This is illustrated by Cox et al. (2000, p. 116).

[...] Action bosses [...] helped members get three crucial aids to re-election: the party endorsement, financial backing, and party and governmental posts. In
return, the bosses received his follower’s support in the LDP presidential election, which he could use either to pursue the party presidency himself or to trade for other positions.

Japanese factions, like their Italian counterparts, have also had an important influence on the distribution of public expenditures. According to Scheiner (2005), p. 807-8, pork barrel spending is targeted to the constituents of strong LDP factions.

[...] funding for local projects is often clearly targeted to LDP Diet members’ financial and political supporters, especially local politicians who deliver the vote for the Diet members (Curtis, 1971; Mulgan, 2000, p. 81; Park, 1998a, 1998b).

2.4 Factions in Mexico’s PRI

Factions in Mexico are called camarillas. They are less formal than Italian or Japanese factions, but they have been highly influential in the PRI, the party that dominated Mexican politics from 1930-2000. Camarillas are based on personal ties of trust across a hierarchy, and members often share some element of their formative or professional life. Camp (2003, p. 104) enumerates “Fifteen characteristics of Mexican Camarillas;” we select the five most relevant to our analysis.

1. The structural basis of the camarilla system is a mentor-disciple relationship

---

8They may share a university advisor, or have been colleagues in a previous position, etc. See Smith (1979) for a detailed study of camarillas.
2. Successful politicians initiate their own camarillas simultaneously with membership in mentor’s camarillas.

3. Every major national figure is the “political child,” “grandchild,” or “great grandchild” of an earlier, nationally known figure.

4. Politicians with kinship camarillas have advantages over peers without them.

5. The larger the camarilla, the more influential its leader and, likewise, his disciples.

The two-way ties between patrons and clients in a camarilla are well-illustrated in the following description of the activities at CONASUPO, a public agricultural support agency.\(^9\) Grindle (1977) writes:

Through a number of high-level appointments, the director of CONASUPO made friends among the leadership of the peasant and middle-class sectors of the party, obligated a number of state governors, developed a following among university students, and established friendships with officials in key government agencies. The extent of the political support he accumulated in this manner made him a valuable member of a political faction whose importance increased as it attempted to influence the selection of the presidential candidate for 1976. If successful in this maneuver, the director could expect to become a close collaborator of the new president. His subordinates were aware of the advantages of “winning” for their own careers. “If he becomes a minister,” commented one respondent, “then

\(^9\) CONASUPO had a broad mandate. See Yunez-Naude (2007, pp. 4-6).

14
his entire equipito [inner circle] will follow him and we’ll all have positions in the Ministry.”

2.5 Factions in China’s CCP

The preceding examples are taken from long-established, (at least) formal democracies. Intra-party factions also operate in systems with less-developed democratic institutions. In China’s Communist Party (CCP) where party politics is largely informal, factions play an important role. In China’s Communist Party (CCP) where party politics is largely informal, factions play an important role. A large literature in Chinese politics studies factions and invariably identifies them as key for understanding political power.

Chinese factions share many traits with their counterparts in Italy, Japan and Mexico. Huang (2000, p. 76), identifies the following five characteristics of factional links in the CCP, many of which resemble those of DC, LDP and PRI factions.

1. The crux of a factional linkage is the exchange of political obligations that concern the well-being of both participants in a hierarchic context.

---

10 “Unlike most Western countries, where formal politics is clearly dominant over informal politics [...] the Chinese informal sector has been historically dominant, with formal politics often providing no more than a facade. Informal politics plays an important part in every organization at every level, but the higher the organization the more important it becomes.” Quoted from Dittmer (1995, pp. 16-17).

11 Huang (2000) writes (p. 1) that “Factionalism, a politics in which informal groups, formed on personalities, compete for dominance within their parent organization, is a well-observed phenomenon in Chinese politics,” and (p. 77) “A leader’s power is essentially based on the strength of his factional networks. The leaders who have the most access to factional networks dominate.”

15
2. It is equally coercive on both participants. Abrogation by either of them can bring about damage or even disaster to both participants.

3. Each participant holds a position of authority at a given level. But direct relations usually exist only between the superior and his immediate inferiors.

4. A factional linkage is not inclusive. Although a leader can develop such linkages with other followers so as to maximize his support, it will be disastrous for a follower to seek multiple linkages with more than one leader. This would give a leader enough reason to suspect his loyalty and hence to withdraw his protection.

5. It can be extended: both ends can be linked to the next higher or lower level of authority in the same fashion.

The goods exchanged across CCP factional linkages are also similar to those in the preceding examples. The superior (patron) rewards the inferior (client) with security or advancement, and is repaid with support.

The prime basis for factions among cadres is the search for career security and the protection of power ... Thus the strength of the Chinese faction is the personal relationship of individuals who, operating in a hierarchical context, create linkage networks that extend upwards in support of particular leaders who are, in turn, looking to their followers to ensure their power. Pye (1981, pp. 7-8).

Like factions of interest elsewhere, CCP factions seek rents from the central government and are thought to affect the distribution of public expenditures. While systematic evidence is
difficult to obtain, at least one study documents this effect. Shih (2004) collected proxies for the factional ties among Chinese politicians and finds that factional ties have an effect on the distribution of bank loans in reform-era China.

2.6 Factions in Chicago’s “Daley Machine”

The Democratic Party in Chicago under mayor and party chairman Richard J. Daley (1955-1976) is a well-studied example of factions operating in a U.S. urban “machine.” During the Daley era, the Chicago Democrats were organized along the administrative lines (wards) of the city in hierarchical networks of clients and patrons. Daley was the party’s chief executive. Beneath him were party committeemen, and beneath them, with some overlap, were alderman – each representing one of 50 wards. Each committeeman appointed a cadre of precinct captains who reported to him. Factional networks also extended into the city government bureaucracy through thousands of patronage jobs controlled by the party. (Guterbock, 1980)

Like their counterparts discussed above, Democratic Party members made exchange across patron-client links; clients at lower ranks delivered votes for their patrons in return for personal promotion and jobs for themselves and their constituents.

“In the heyday of the machine during the Daley years ... jobs were allocated to ward and township committeemen in proportion to the individual committeeman’s influence and the number of votes his ward delivered for machine candidates. [...] Generally, the committeemen parceled out the jobs they “owned” to their precinct captains on the basis of the captains’ ability to garner votes.
If a captain failed to deliver his precinct, he could be “viced” or fired from his job. If his failure were less serious, he might only lose some of the jobs under his control.” Quoted from Freedman (1994, p. 39).

In addition to patronage jobs, party factions directed public resources to themselves and to their constituents by means of their control over city and county bureaucracies. A city attorney and precinct captain explained how, in exchange for votes, he worked to provide better public services and, indeed, lower taxes for his constituents.

“I consider myself a social worker for my precinct. I help my people get relief and driveway permits. I help them on unfair parking fines and property assessments. The last is most effective in my neighborhood [middle class]. The only return I ask is that they register and vote.”

Overall, the party and its internal politics, more than the formal offices of government, determined public spending:

“It was through [Daley’s] control of the party, not his elective office, that he gained complete control of the city council ... Thus the mayor, not the council, decided the budget; the mayor, not the council, really decided on the legislation that ran the city.” Allswang (1986, p. 143).

---

12 Quoted in Allswang (1986), page 141.
2.7 Summary: Defining Traits of Factions of Interest

These examples of factions, and others from various times and places around the world,\textsuperscript{13} present several common traits upon which our model is based:

1. Factions of interest are hierarchical networks of party members.

2. A faction member transacts mostly with his direct hierarchical superior (patron-client relationship). The patron expects to be supported in his ascent to power. In return, the patron gives the client resources that help advance (or at least secure) the client’s position in the hierarchy.

3. Factions of interest do not typically coalesce around ideological or policy positions. Instead, they are devoted primarily to the capture of public resources.

4. The existence of factions results in an allocation of resources that follows a factional logic, not necessarily the welfare of the party as a whole, or any efficiency criterion.

Along some dimensions, we see variation across the examples. The formality of the faction, for example, ranges from high (Italy, Japan) to low (China). The system of factional competition may be operated centrally almost as an incentive scheme (Chicago, by Daley), or it may be the result of informal self-organization of competing groups (Mexico). The model in the following sections is sufficiently general that it need not take a stand on these dimensions.

\textsuperscript{13}Other, well-studied examples of factional politics include, New York City’s Tammany Hall (Riordon, 2004), India’s Congress Party (Kohli, 1990), and the “Shadow State” of Sierra Leone (Reno, 1995).
3 Model

We are primarily interested in the effect of factions on the allocation of public resources. To study these effects we propose, as a starting point, an exceedingly simple view of the faction: a faction is a team of politicians who are mutually dependent on each other for career advancement. This simple model generates number of implications for the allocation of public resources, which are derived in Section 5. Because the model has some non-standard elements, we devote the final subsection to discussing its assumptions and formally investigate the consequences of relaxing several of them in Section 6.

3.1 Setup

Time is discrete and indexed by $t$. There are $S$ states (localities), indexed by $s$; in each of them an election takes place in every period. Two national parties compete in each election. For expositional ease we will focus our analysis on the factions of one of these two parties. In Section 6.7 we show how to extend our analysis to the case of two or more factionalized parties competing with each other.

3.2 The Party and Its Officers

A party is a series of positions that party officers wish to hold. Positions are characterized by their rank. Rank $r + 1$ is senior to rank $r$, and $r = 0$ denotes the lowest possible rank. A party’s candidate in the state $s$ election holds rank 0 in the party. There is no maximum
rank, and the number of positions of each rank is implicitly determined by the promotion policy, modeled below.

These positions may be thought of as posts in the party bureaucracy, or they could be administrative positions in ministries or state enterprises if the party has power of patronage. The different ranks need not be formal, with distinct titles and authorities. Rather, the ranking is meant to capture, more broadly, the path by which an officer’s career advances.

Party officers belong to different states. An officer from state $s$ can exert effort $e \in [0, 1]$ which increases the probability that a public project is provided to state $s$. You can imagine that officers were born in that state, and their special knowledge of that state makes their effort specialized. Exerting effort $e$ costs the faction member $c(e)$. The cost function $c(\cdot)$ is assumed to be convex, and $c(0) = c'(0) = 0$.

Effort has several interpretations, not mutually exclusive. First, effort may represent investment in a lobbying process by which faction members compete to divert public resources toward their state. Second, effort may represent fund raising activity on the part of faction members. Finally, effort may capture the degree to which the officer resists the temptation to skim public funds allocated to state $s$.

In this basic model, the party officers’ objective function is myopic: they simply derive a given amount of utility (which we normalize to 1) from being promoted to the next rank at the end of each period. In Section 6.2 we extend the model to allow for forward-looking officers.
3.3 Factions, Recruiting, and Promotions

Party officers of all ranks are partitioned into factions, according to the state to which they belong. State $s$ at time $t$ has a *faction* of size $S_t \geq 0$, which is composed of all party members who belong to state $s$.

Promotions are made at the end of period $t$. In the promotion process, all members of the state-$s$ faction share the same fate: either the party won election in state $s$, and then each faction member is promoted up one rank in period $t + 1$; or the party lost the election, and then all members of that faction are out of the party from $t + 1$ onward.\(^{14}\) If the party wins the state-$s$ election at time $t$ then a new rank-0 member joins the faction and runs for election at time $t + 1$. Thus the promotion system is up-or-out. Section 3.6.4 discusses which features of this promotion process are essential for our analysis.

We can interpret this promotion rule as a reduced form capturing internal party politics in a “bottom up,” or representative democracy system. Suppose that, in order to be promoted in internal party elections, every party member (patron) needs the support of at least one member in the next lower echelon (client). Suppose further that faction members vote for each other. Now, if the lowest ranking member of a faction fails to advance because he loses the election then there is no-one to support the rank-1 member of the faction, who then also fails, and so on. Thus the advancement of the entire faction turns on the outcome of the election.\(^{15}\)

\(^{14}\) In Section 6.2 we extend this basic model to allow for more resilient factions that, even if they lose an election, can always return to compete in the next election.

\(^{15}\) This “internal democracy” interpretation is developed formally in an earlier version of this paper (Persico
Notice that this promotion rule links the evolution of a state’s faction to the outcome of the election in that state. If the party wins the state-s election in $t$, then $S_{t+1} = S_t + 1$ (the increase in the size of the faction reflects the fact that a new rank-0 officer has joined the faction). If the party is defeated then $S_{t+1} = 0$. If the party wins a state $s$ which was previously controlled by the opposition, then $S_{t+1} = 1$.

### 3.4 Elections and Public Projects

In each state and in every period there is an election in which the party candidate (the rank-0 officer) runs. We abstract from the details of this election simply assume that the party candidate is more likely to win the election if his state receives an indivisible unit of public project before the election.\(^{16}\) In state $s$ the probability of electing the party candidate increases from $b_s$ to $b_s + \Delta_s$ when the public project is provided.

The probability that the public project is provided to state $s$ depends on the sum of efforts devoted by party officers to that state. Let $e_r$ denote the sum of all efforts directed by officers of rank $r$ to region $s$. Then state $s$ receives a public project with probability

$$\Pr (g = 1) = (1 - \alpha) \sum_{r=0}^{\infty} (\alpha)^r e_r$$

or 1, whichever is smaller.

Equation 1 implies that the effect of an officer’s effort on the probability of winning is linear\(^{16}\)In an earlier version of this paper (Persico et al., 2009) we offer a microfoundation for this assumption in the form of a model where rational voters interpret the pre-electoral receipt of the public project as a signal.
and independent of both his party’s baseline support, $b_s$, and the efforts of his fellow faction members. In Section 6.8 we investigate the consequences of relaxing these assumptions. For technical convenience we will assume that the scalar $\alpha$ is less than one. This guarantees that the summation (1) converges and implies that the effort of higher-ranking officers has less impact on the provision of public resources. In Section 6.5 we discuss how to extend the analysis to the case where $\alpha > 1$. As we explain in Section 3.6, we need not take a stand here on whether the effort exerted in favor of state $s$ is rival to that exerted for other states.

We will call states with high $b_s$ party “strongholds,” because they are likely to vote for the party regardless of whether they receive the public good. States with high $\Delta_s$ are called “swing” states because there is a high probability that providing the public good will change the election outcome in these states. We assume for convenience that $b_s + \Delta_s < 1$; that is, the party can never be 100% sure of winning the election in any state.

### 3.5 Timeline

At time $t$,

- the $S_t$ members of the state $s$ faction choose effort $e_{it}$
- the public good $g_t$ is realized according to the probability distribution (1)
- the election takes place and the party either wins or loses in state $s$
- promotions are made and $S_{t+1}$ is determined.
3.6 Discussion of Modeling Assumptions

In this section we discuss what assumptions play an essential role in our model. Because the model is novel some of its assumptions are, inevitably, unconventional. Many of these modeling assumptions are made for tractability and could easily be modified. In general, the plausibility/appeal of the assumptions should be judged in light of the model’s primary purpose: to build on the qualitative evidence provided in Section 2 and describe a plausible and testable causal mechanism for certain patterns in public goods allocation.

3.6.1 The Faction

Since the per-period survival probability of a state-s faction is bounded above by $b_s + \Delta_s < 1$, every faction will die in finite time. The promotion process described in Section 3.3 guarantees that all factions born after time 0 will share the following properties: (a) all factions will have exactly 1 member per rank between rank 0 and rank $S_t$; (b) in every period, a faction will either grow by one member or else collapse. Figure 1 depicts an example of the evolution of factions between period $t$ and $t + 1$. In this example, the faction in state 3 collapses, while the others grow by 1 member.

We made several stark assumptions about the nature of factions. First, we tied each faction to a state. Second, we made the faction a purely vertical (and exclusive) network; only past rank-0 members can be part of the faction. Third, factions do not overlap — an officer...

---

17 As noted above, the state could easily be replaced by any well-defined constituency such as labor unions, public employees, military personnel, agrigultural workers, etc.
cannot belong to more than one faction. Fourth, there is no maximum rank in the party. Fifth, there is no fixed number of positions in the party, and thus no explicit contest among factions for positions. Each of these assumptions was made for simplicity of exposition and they could be relaxed considerably without much affecting what we are interested in, that is, the implications for public spending (Proposition 3, particularly a.-c.). What will be crucial for our analysis is that faction members behave as a team, mutually dependent on each other for career advancement. Section 6.4 shows this formally.

### 3.6.2 Competition Among Factions

We need not take a stand here on whether the effort exerted in favor of state \( s \) is *rival* to that exerted in favor of other states. The relevant public resources may come from a fixed pool that could be allocated to any state (effort is rival), or they may come from a pool that is
only available to state $s$ (effort is non-rival). One might be concerned that the interpretation of rival effort is not proper here, because the probability (1) does not depend on the effort for states other than $s$; but the rivalry interpretation is proper. Expression (1) can be recovered as the limit probability of winning a prize in a tournament in which $N$ factions compete for $qN$ prizes ($q < 1$), when the number of competing factions $N$ becomes large. So expression (1) does not preclude the interpretation of factions competing for a fixed amount of public spending. For the details of this argument see Appendix A.1.

3.6.3 Voters

Voter behavior enters the model in reduced form. In a previous version of the paper (Persico et al., 2009) we show that this reduced-form model can in fact be derived from a model of rational voters who interpret the pre-electoral receipt of the public project as a signal of the power of their faction.

3.6.4 Promotion Policy

We make two distinctive assumptions about the party’s promotion policy: It is both “up or out” and “bottom up;” either the faction’s lowest rank member gets elected and the entire faction is promoted one rank, or else the entire faction fails. As we show in Section 6.2, the up-or-out assumption is not essential to the results. Our primary results go through with more resilient factions. What is key is that the faction grows more powerful when it wins elections. That said, there are real-world cases, such as Mexico, in which political careers
are effectively up-or-out.

The “bottom up” feature may appear more important for our results, but this is misleading. For example, we could have developed a model in which the faction is “pulled from above,” say by its chief, rather than pushed from below. The mechanics would be somewhat different, but our model’s key feature would be maintained; even in this “pull from above” model all faction members would exert effort for the common good of the faction (in this case the good of the chief). As long as this effort increases local public goods provision, the kinds of correlations collected in Proposition 3, particularly a.-c., would obtain in this “pull from above” model too. So, what is key is not the bottom-up structure of promotions, but rather the “common enterprise” nature of incentives. Again, this argument is made formal in Section 6.4. We view these incentives as deriving from internal party rules which promote faction-building by providing career benefits to individuals who band together in informal groups. That said, even though the “bottom up” assumption is not critical for our results, it is not at all a bad assumption: in many parties promotions require a strong element of support from below, owing partly to a formal process of representative democracy within the party, where officers are selected for assemblies of different ranks, and the selectorate of the rank $r$ assembly is rank $r - 1$ assembly.
4 A Special Case: Unitary Party Benchmark

In the standard, unitary-party model, a given budget is allocated across localities to maximize the sum of the probabilities of winning.\footnote{Considering other objectives for the party, such as winning a majority of the districts, would not qualitatively change the results.} (See e.g. Lindbeck and Weibull 1987). Our analysis nests as a special case the allocation implemented in that model.

We obtain the standard allocation by restricting $\alpha = 0$. Under this assumption, power is not distributed across the party hierarchy: only the effort exerted by rank-0 officers matters for procuring public resources. Let us therefore focus on the behavior of these officers. The rank-0 officer in state $s$ chooses $e$ to maximize the probability of winning the election minus the cost of effort,

$$\max_{e} b_s + \Delta_s \cdot e - c(e).$$

The optimal effort level $e^*_s$ therefore solves

$$c'(e^*_s) = \Delta_s.$$ 

In this allocation, swing localities receive resources in proportion to their responsiveness ($\Delta_s$); and the baseline level of support for the party ($b_s$) does not affect the allocation. These properties of the resource allocation are the hallmarks of the standard models of distributive politics.

In our specific setup, the unitary party paradigm has even stronger predictions, because the return to allocating resources to a locality is linear (with slope $\Delta_s$). This implies that, in a
unitary party, resources would be allocated maximally \( (e = 1) \) to all localities with \( \Delta_s \) larger than a threshold, and no resources would go to the other localities. This allocation, too, can be achieved in our model by restricting the cost function \( c(\cdot) \) to be linear.\(^{19}\)

Thus we see that our analysis nests as a special case the allocation that is implemented in the conventional unitary-party models of distributive politics. In this special case \( \alpha = 0 \); that is, power is not distributed in the party organization. In what follows we study the case when power is distributed, that is, \( \alpha > 0 \).

5 Resource Allocation in the Presence of Factions

We now turn to characterizing the allocation of resources that emerges when power is distributed across the party hierarchy. Towards this end, we first establish some properties of the equilibrium size and effort of factions. In what follows we omit the state index \( s \) when no confusion can arise.

5.1 Definition of Equilibrium

Since we assume that party officers have myopic objectives, their equilibrium behavior is given by a sequence of Nash equilibria of the stage game outlined in Section 3.5.

Some care must be taken with initial conditions. At time 0, we can allow factions with

\(^{19}\)The slope of the linear function \( c(\cdot) \) corresponds to the shadow price of resources in the optimal allocation for the unitary party model.
more than one member at any rank. But, no matter what the time-0 structure is, all
factions born after time 0 will have exactly 1 member per rank between rank 0 and rank \( S_t \)
(see the discussion at the beginning of Section 3.6.1). Moreover, since per-period survival
probabilities are always strictly less than one, in finite time all factions will be born after time
0. Thus, in the long run, initial conditions do not matter. We therefore focus on equilibria
where at all times all factions have exactly 1 member per rank between rank 0 and rank \( S_t \).
We call this a long-run (Nash) equilibrium.

5.2 Faction Effort For Given Faction Size

Because within each state \( s \) at time \( t \) the party has a faction with exactly one member per
rank, we may identify a faction member with his rank \( r \). Let \( R \geq 0 \) denote the number of
faction members. Member \( r \) solves

\[
\max_{e_r} b + \Delta \Pr (g_t = 1) - c(e_r) \\
= \max_{e_r} b + \Delta \left[ (1 - \alpha) \sum_{r=0}^{R} \alpha^r e_r \right] - c(e_r).
\]  

(2)

The equilibrium level of effort \( e_r^* \) solves

\[
c' (e_r^*) = \Delta (1 - \alpha) \alpha^r.
\]  

(3)

The effort of a faction member is therefore increasing in \( \Delta \) and does not depend on \( b \). Also,
equation (3) does not depend on \( R \), so member \( r \) will put in effort \( e_r^* \) independent of his
faction’s size. Therefore, the total effort put forth by a faction is increasing in the faction’s
size. We summarize these observations in the following proposition.
Proposition 1 In a long-run Nash equilibrium the effort of a faction member is increasing in $\Delta$ and does not depend on $b$. The total effort of a faction, and thus its probability of survival, is increasing in its size.

5.3 Steady State Distribution of Faction Size

Some aspects of the equilibrium of our game will depend on the size of factions at time zero. However, the effect of these initial conditions dissipates with time. Over time, then, one can ignore the effect of initial conditions and focus on the steady-state properties of the equilibrium. In this section we characterize the steady-state distribution of faction size. In a long-run Nash equilibrium the probability of a faction being of size $R + 1$ in period $t + 1$ equals the probability of being size $R$ in period $t$ times the transition probability. Formally,

$$\pi_{t+1}(R + 1) = \pi_t(R) \cdot \left[ b + \Delta (1 - \alpha) \sum_{r=0}^{R} (\alpha)^r e_r^* \right].$$

At a stationary equilibrium $\pi_t(\cdot) = \pi(\cdot)$, so the stationary distribution can be characterized by the following difference equation:

$$\pi(R + 1) = \pi(R) \cdot \left[ b + \Delta (1 - \alpha) \sum_{r=0}^{R} (\alpha)^r e_r^* \right] \tag{4}$$

$$\pi(0) = 1 - \sum_{k=1}^{\infty} \pi(k).$$

Since by assumption $b + \Delta < 1$, we have that $\pi(R) > \pi(R + 1)$ for all $R$. Figure 2 provides a qualitative picture of the stationary distribution of faction size for a given pair $\Delta, b$.

We now show that swing states, and states with a large base of support for the party, are more likely to have large factions.
Figure 2: Steady-state distribution $\pi$ of faction size in state $s$.

**Proposition 2** Increasing $\Delta$ and/or $b$ results in a first-order stochastically dominant shift of the steady-state distribution of faction size.

**Proof.** Suppose $\Delta$ increases. Then by equation (3), $e^*_r$ increases for all $r$. From equation (4), then, the new steady-state size distribution $\pi'$ has the property that

$$
\frac{\pi'(R+1)}{\pi'(R)} > \frac{\pi(R+1)}{\pi(R)}.
$$

(5)

It cannot be that $\pi'(0) \geq \pi(0)$, because then we would have $\pi'(R) > \pi(R)$ for all $R > 0$ and then both distributions $\pi$ and $\pi'$ could not sum to 1. So it must be $\pi'(0) < \pi(0)$, and then equation (5) implies that there is a unique value $\overline{R}$ such that $\pi'(R) < \pi(R)$ if and only if $R < \overline{R}$. This establishes that $\pi'$ first-order stochastically dominates $\pi$.

If $b$ increases, $e^*_r$ does not change for any $r$, and equation (5) again holds. The previous reasoning then proves the result. ■

33
5.4 Resource Allocation

This section establishes three main points. First, in equilibrium the allocation of resources reflects the power of the faction. Second, and related, there is a systematic bias in favor of party strongholds. Third, factions generate persistence in the resource allocation. The next proposition makes these points and moreover, in points c.-e., it draws out several additional implications for the allocation of public resources.

**Proposition 3 (Allocation of resources)** The steady-state probability that a state receives public resources depends on the size of its faction. Through this channel the following results arise in our model:

a. In steady state, swing states (higher $\Delta$) and party strongholds (higher $b$) are more likely to receive public resources from the party.

b. In steady state, given two states with the same $b$ and $\Delta$, the state with a longer spell of uninterrupted electoral success for the party is more likely to receive public resources from the party.

c. The probability that a state receives public resources from the party at time $t$ is predicted by the future success within the party of the officer who holds rank 0 at time $t$.

d. Conditional on winning election at time $t$, the vote-getting ability of a rank-0 officer is uncorrelated with the probability that his constituents receive public resources from the party in the future.

e. Conditioning on faction size at time $t$ eliminates all the effects described in parts a.-d.,
except for the effect of $\Delta$ in part a. States that are dominated by the opposition (faction size at time $t$ is equal zero) receive no resources from the party at time $t$.

**Proof.** According to Proposition 1, the probability that a state receives the public project given faction size $R$ is an increasing function of $R$. This proves the introductory statement.

Proving part a. requires averaging out faction size. The probability that a state receives the public project conditional on faction size $R$ is an increasing function of $R$. Taking an average of this function using the steady-state distribution of $R$ yields the probability that a state receives the public project. By Proposition 2, that distribution is stochastically increasing in $b$. Thus states with higher $b$ have a higher probability of receiving the public project. The same argument applies to states with larger $\Delta$, and in addition factions in those states will exert more effort (Proposition 1), which establishes the result for those states.

Proof of part b. is immediate.

To prove part c., let $B = b + \Delta$ and

$$P_\tau = (\text{party wins at } t + 1, \ldots, \tau | \text{party wins at } t).$$
Then we can write

\[
\Pr(g_t = 1 | \text{outgoing rank-0 officer at } t \text{ promoted through } \tau)
\]

\[
= \Pr(g_t = 1 | \text{party wins at } t, t+1, \ldots, \tau)
\]

\[
= \frac{\Pr(\text{party wins at } t, t+1, \ldots, \tau | g_t = 1) \cdot \Pr(g_t = 1)}{\Pr(\text{party wins at } t, t+1, \ldots, \tau)}
\]

\[
= \frac{P_\tau \cdot \Pr(\text{party wins at } t | g_t = 1) \cdot \Pr(g_t = 1)}{B \Pr(g_t = 1)}
\]

\[
> \frac{[(1 - B) + B (1 - P_\tau)] \Pr(g_t = 1)}{[(1 - B) + B (1 - P_\tau)] \Pr(g_t = 1) + [(1 - b) + b (1 - P_\tau)] \Pr(g_t = 0)}
\]

\[
= \Pr(g_t = 1 | \text{outgoing rank-0 officer at } t \text{ not promoted through } \tau)
\]

(6)

The inequality follows from algebraic manipulation.

Part d. Regardless the politician’s ability to attract votes when running for office, conditional on having been elected, in our model his vote-getting ability is irrelevant for his future role in the life of the faction. In particular, the state of a rank-0 officer that barely managed to get elected is just as likely to receive public goods as one with an officer that was elected by a large margin.

Part e. Immediate. ■

Part b. of the above proposition indicates that the resource allocation is persistent. States with a longer spell of uninterrupted electoral success for the party are more likely to receive public resources. This is because such states have larger factions. By the same token, failure to receive resources is also persistent, because it makes it more likely that the faction is eliminated. Of note, the persistence is associated to the state (or locality) but not to the
politician. The politician does not, in our model, persist in office.\footnote{This is important for making predictions about political systems, such as Mexico’s, where there is a high degree of turnover in the positions of the ruling party’s hierarchy and yet substantial persistence and stronghold spending. See, e.g., Estevez, et al. (2006) and a previous version of this paper Persico et al. (2007).}

Part c. of Proposition 3 illustrates one way in which the career paths of politicians are linked to the allocation of public resources. Like part d. of the proposition, it makes new predictions about the relationship between political careers and public spending that, to our knowledge, have been little explored. We note, however, that the stark no-correlation result obtained in part d. is a consequence of the assumption that party members run for office only once. Were we to allow an outgoing rank-0 officer to run for office again, we would likely observe some correlation.

6 Discussion and Extensions

In the preceding analysis we used an especially simple model to study the effects of factions on the allocation of public resources. In this section we discuss aspects of that model further as we extend it along a number of dimensions. Here our goal is to understand better the robustness of the simple model’s predictions and to investigate what factors determine the degree to which a party is influenced by factions of interest and benefits from them.
6.1 The Distinction Between Faction and Party

We begin with consideration of the conceptual difference in our model between a faction and a party. Is a faction functionally equivalent to a party? We can conceive of at least two ways of conceptualizing this broad question. We address them in turn.

First, if factions are different from parties then why do factions choose to be? Why, that is, don’t factions split off and form separate parties? Our answer to this question is that a faction competes under importantly different rules from a party. While the party faces competition (from other parties) in the state-s election, in our model the state-s faction does not face internal competition to field a candidate in that election. This is because in our simple model there are no primaries where different factions can contest the nomination—we assume that the rank-0 candidate is automatically a faction member. In this sense, the faction benefits from staying within a party, so long as internal party rules protect its power to nominate. Were the faction to split off from the party, its candidate would lose that protection and would have to compete in a three-way race instead of a two-way race. In Section 6.3, below, we discuss the effect of primaries and argue that the ability to control nominations is crucial to the existence of intra-party factions.

A second conception of the functional distinctions between faction and party considers the question: how is a faction affected by being in a party? Party power may affect faction power and incentives. In a democracy, for example, the majority party in the assembly often has an advantage in the allocation of public funds (see Alouby 2009 for evidence of this in the U.S.). The results from our simple model can be extended to gain insights about a more
sophisticated model where the faction’s power to provide public goods depends on its party’s strength. We sketch one such model here.\textsuperscript{21}

Suppose the party \emph{controls} a state when the party has a faction member of rank 1 or higher in that state. And suppose that if the party controls more than $S/2$ states then the party has the majority of the seats in the assembly. In that case, the return to faction members efforts is $\Delta_M$. If the party controls $S/2$ or fewer states then the party is in the minority and the return to effort is $\Delta_m < \Delta_M$. The wedge between $\Delta_m$ and $\Delta_M$ captures the majority advantage in the provision of public goods. The sharp threshold $S/2$ is meant to capture a two-party majoritarian system; it could be generalized to capture other political systems.

The factional dynamics in such a model follow directly from equation (3): when the party holds a majority, a faction member’s equilibrium effort solves (3) with $\Delta = \Delta_M$; when the party is in the minority, a faction member’s equilibrium effort solves (3) with $\Delta = \Delta_m$. Clearly effort is lower when the party is in minority status, which means that minority party factions will be less durable and the party will be less likely to grow the number of seats it controls. This model thus produces persistence in majority status: the probability that a party keeps its majority status is higher than the probability that a minority party gains majority status.

Like in the baseline model, in this model an increase the stronghold parameter $b_s$ does not change the equilibrium effort put in by any faction, \emph{given party status}. However, increasing $b_s$ makes it more likely that the party controls state $s$, and so it increases the likelihood

\textsuperscript{21}We are grateful to an anonymous referee for suggesting this line of reasoning.
that the party achieves and retains majority status; and when party status switches between majority and minority effort does change. Thus, in this model increasing the stronghold parameter in a given state can have spillovers on the effort of other factions—it can lead them to increase their effort, which in turn benefits faction $s$. This indirect channel thus reinforces the positive effect that increasing $b_s$ has on the probability that state $s$ receives local public goods.

6.2 Forward-Looking Officials and Resilient Factions

Our baseline model assumes party officials are myopic and that factions are relatively fragile; one lost election means that the faction disappears. A natural question is whether the team incentives at the core of that model break down when officials are forward-looking and know that one loss won’t devastate the faction. We address this question here by extending the model to allow forward looking officials and resilient factions.

Suppose, as in the baseline model, that every member of a faction that wins its election gets spoils of office equal to 1, that a winning faction grows by one member, and that all members move up by one rank. Assume, in addition and different from the baseline model, that every member of a faction that loses its election gets no spoils of office (zero) in that period, but does not disappear. Suppose, instead, the faction stays the same size, each member maintains his rank, and the faction survives to compete in the next election. Finally, assume that all this happens in each period with probability $p$; with probability $1 - p$, the faction shrinks to zero before any effort can be made in that period. This technical assumption will
ensure that faction size does not explode. The probability $1 - p$ might be interpreted as the likelihood of a scandal or accident that devastates the faction.

In this extended model, the value of being of rank $r$ in a faction of size $n$ is given by $V(r, n)$, where this value function solves the following equation:

$$V(r, n) = p \left\{ \max_{e_r} \Pr(E|n, e^*_x, e_r) \left[ 1 + \beta V(r+1, n+1) \right] + \left[ 1 - \Pr(E|n, e^*_x, e_r) \right] \beta V(r, n) - c(e_r) \right\},$$

and $\Pr(E|n, e^*_x, e_r)$ denotes the probability of winning the election in a faction with $n$ members where member $r$ exerts effort $e_r$ and all other members exert effort according to the equilibrium strategy $e^*_x$.

We simplify the solution to (7) by assuming that the effort of each faction member has the same impact on the probability of faction survival. This assumption allows us to focus on symmetric equilibria in which each faction member exerts the same effort. In this case the value function is independent of the faction member’s rank. A drawback of this assumption is that the per-period probability of faction survival (now $p \cdot \left[ b + \Delta \sum_{r=0}^n e_r \right]$) could exceed 1. If, however, we make the cost of effort large enough we can ensure that this event happens with arbitrarily small probability.

To summarize, if the effort of each faction member has the same impact on the probability of faction survival, we can rewrite equation (7) as

$$\left( \frac{1}{p} - \beta \right) V(n) = \max_{e_r} \left[ b + \Delta \sum_{r=0}^n e_r \right] \left[ 1 + \beta (V(n+1) - V(n)) \right] - c(e_r).$$

We now make the following technical assumption, which is sufficient to guarantee existence
of a solution to equation (8).

**Assumption 6.2:** The cost of effort is convex with \( c'(0) = 0 \) and \( c'(X) = \infty \) for some \( X < \left( \frac{1}{p^3} - 1 \right)^{\frac{1}{3}} \).

**Proposition 4** Suppose Assumption 6.2 is satisfied. Then there exists a solution to equation (8) such that:

1. \( V(n) = K_1 + K_2n \).

2. The equilibrium effort of a faction member \( e^*_n \) is independent of the size of the faction \( n \) and of the size of the stronghold effect \( b \).

3. The equilibrium probability that a faction of size \( n \) grows is increasing in \( n \) and in \( b \).

4. The long run density distribution of faction size is decreasing in size.

5. Finally, increasing \( b \) produces a stochastic-dominance shift in the long run distribution of faction size. Thus localities with a higher \( b \) will have a higher average probability of receiving the public good.

**Proof.** See Appendix. ■

Proposition 4 shows that the signature predictions from our basic model are robust to having forward-looking agents and resilient factions. Larger factions still exert more effort and are, thus, still more likely to deliver public goods to their constituencies and win elections. Also, party strongholds will still tend to produce larger factions. Thus, the predictions of policy persistence and stronghold spending carry over to this setting.
It is worth noting that assumption 6.2 is sufficient, but not necessary, to guarantee existence of solution to 8 that is an affine function of \( n \). Indeed, we have derived a closed-form solution to 8 for the case of quadratic costs of effort.\(^\text{22}\)

### 6.3 Obstacles to Factional Politics and the U.S. Case

As we consider the importance of factions for public spending, it is notable that national parties in the U.S. do not have prominent factions of interest.\(^\text{23}\) Why is that? And more generally, what determines the degree to which a party is divided into factions of interest? We address these questions next.

**Party Dominance** Electorally dominant parties are more likely to develop factions. It is no coincidence that all the parties mentioned in Section 2 have been in power for long spells—many decades. Dominance is likely to breed factions for several related reasons. First, from the point of view of a party officer, holding rank \( r \) is more valuable if a party is now in power, is likely to be in power soon, and is likely to be in power in the future. In this sense, the rewards that induce factional behavior are more powerful in dominant parties. Second, and related, a party that is in office for an extended period is able to penetrate government bureaucracies. In this way, non-political positions in state enterprises, public administration, regulated businesses, etc., become part of the party reward system—ranks, in

\[^{22}\text{In that case, if } c(e) = e \cdot \frac{e^2}{2} \text{ then } V(n) = K_1 + K_2 n \text{ where } K_1 = b \frac{1+\beta K_2}{p-\beta} \text{ and } K_2 \text{ solves } \beta^2 K_2 + \left(2\beta - \left(\frac{1}{p} - \beta\right) \frac{e^2}{2}\right) K_2 + 1 = 0.\]

\[^{23}\text{Factions of principle are, however, common within national parties. The Republican Party, for example, is divided into Reagan Republicans, Rockefeller Republicans, the Religious Right, etc.}\]
our terminology. Third, the negative consequences of factional organization on the resource allocation (as spending is diverted from swing states) are less important if a dominant party has less fear of electoral competition.

These observations may partly explain why factions of interest are relatively rare in U.S. national parties, which tend to alternate in power. In contrast, factions can be found in the state Democratic parties in the post-civil-war South (see Key 1949) and in urban political machines, both of which continuously held power for extended periods of time.

Control over Nominations In Section 6.1, we suggested that the strength of factions derives importantly from their control over nominations. Depending on both party and legal rules, factions may have more or less ability to choose new party officers and candidates for election. Nomination control is key, we argue, for faction strength. To see this, recall from Section 3.3 the simple story of internal party politics that motivated the up-or-out promotion rules we assumed in the basic model. In that story, promotion in internal party elections requires that every party member (patron) have the support of the member in the next lower echelon (his client) and faction members vote for each other in sequence, starting from the lowest rank. If the lowest ranking member of a faction fails to win election then there is no one to support the rank-1 member of the faction, who then also fails, and so on. Thus the loyalty of 0-rank candidates is highly valued by fellow faction members, because disloyal candidates reduce their likelihood of being promoted. If recruitment is controlled by the faction, then, we should expect high-loyalty candidates to be picked. By the same token,
the long-term viability of factions also depends on their ability to control nominations.\textsuperscript{24} Consistent with this view, in Italy’s DC and Japan’s LDP, factions effectively had control over nominations and jealously guarded it.\textsuperscript{25} Conversely in the U.S., where due to legal constraints nominations are usually decided in primaries, factions are not overly strong.\textsuperscript{26}

**The U.S. Case** When considering the role of factions in U.S. party politics, it is important to distinguish between national parties and state and local party organizations. We mentioned above that neither national party is dominant in the U.S., which makes it is less likely that factions would develop in national party organizations. There are other reasons,

\textsuperscript{24}If some other entity—the president, or the public—nominates candidates, those candidate are likely to be loyal to those entities. On this point, see Cox et al. (1999).

\textsuperscript{25}Regarding Italy’s DC, Zuckerman (1975, p. 33) writes:

> It would seem that in regions where a national faction leader is present other political patrons or aspiring patrons will associate with his faction. [...] In regions where there are two patrons seeking national prominence, each will associate with a different national faction.

In Japan, particularly before the 1994 reform, nominations were decided in national-level negotiations in Tokyo. Cox et al. (1999, p. 40) write:

> The factions competed just as fiercely over endorsements as they did over posts, seeking both to secure nominations for their own non-incumbents and to protect their incumbents from the appearance of endorsed non-incumbents in their districts.

\textsuperscript{26}In the US, due largely to legal constraints, national parties have relatively little say in the nomination for congress. Instead, primaries typically devolve that power to the mass of party members. (Katz and Kolodny 1994, p. 31). Concerning the weakening effect of primaries on party discipline, see generally V. O. Key (1958), Ch. 14.
too. From the perspective of career concerns, national parties are numerically small and relatively uninfluential in the U.S. compared to state parties. This makes national parties less appealing as a target for politicians intent on building networks. In addition, U.S. national parties have peculiar institutions at the national level whereby committee chairmanships, which confer great power of patronage, are assigned by seniority. Thus, access to these powerful posts does not require politicians to marshal the support of other party members. For all these reasons, factional politics does not develop at the national party level. If we are looking for factions of interest, therefore, we must look in the state parties and at the local level. Again, this is indeed where factions are found, for example in local party machines, or in the state parties of the U.S. south.

6.4 Benefits of Factional Politics: Incentivizing Team Effort

Section 6.3 suggests that faction members will be chosen for their loyalty to the faction, not for their loyalty to the party. Why then might parties seek to support factional ties? In our baseline model of factions, the careers of all faction members, even members who are not on the ballot, are tied tightly to the outcome of elections. In this section we show how these ties, and the incentives they create for team effort, benefit the party; with greater team effort, the party wins more elections. To show this, we extend the model to allow variation in the strength of such team incentives. We will find that when a party’s internal incentives

\[ \text{In part this may be due to the federal organization of the U.S government. — for example, elected positions at the federal level, while often very important, are only 600, compared to more than 500,000 elected positions at the state and local level (Katz and Kolodny (1994), p. 27.)} \]
place greater weight on factional ties, the party supports more powerful factions and, thus, wins more elections. In this sense, then, factional politics is beneficial for the party.

6.4.1 Two Forms of Effort

In this extension, a party official can exert two different forms of effort. The first form, $e^1$, has the same effect as effort in the baseline model: it increases the probability that the officer’s state receives the public good and, therefore, elects the party’s candidate. The second form of effort, $e^2$, benefits only the officer himself (it generates private spoils of office). By varying the “weight” of the private spoils of office in the officer’s maximization problem we can capture, in a very flexible way, the relative strength of team (as opposed to private) incentives.

The rank-$r$ officer solves

$$
\max_{e^1_r, e^2_r} \left\{ b + \Delta \left[ (1 - \alpha) \sum_{r=0}^{R} \alpha^r e^1_r \right] + \sigma_r (e^2_r) - c (e^1_r + e^2_r) \right\}
$$

where $\sigma_r (e^2_r)$ gives the value of private spoils and $c (e^1_r + e^2_r)$ is the cost of effort expended in either form. The marginal benefit of effort expended on private spoils, $\sigma' (\cdot)$, relative the marginal benefit of effort expended on behalf of the team, $\Delta (1 - \alpha) \alpha^r$, captures the relative strength of team incentives. We will assume $\sigma (\cdot)$ is concave and $\sigma' (0) = \infty$.

The central tradeoff for the officer derives from the fact that the two forms of effort are substitutes in terms of cost but not in terms of benefit. The first form of effort only benefits the officer (and the rest of the faction) if his party’s candidate wins. The second form of
effort benefits the officer regardless of the election outcome; the private spoils are a non-team benefit of the officer’s effort. The following proposition describes the effects of changes in the strength of team incentives on the effort and influence of factions.

Proposition 5 1. There exists a unique solution $e_{r}^{1*}, e_{r}^{2*}$ to member $r$’s maximization problem. The solution is such that $e_{r}^{2*} \in (0,1)$ and $e_{r}^{1*} < 1$.

2. If $\sigma_{r}(\cdot)$ is independent of $r$ then type-1 equilibrium effort decreases in $r$ and is zero for some $r$ large enough.

3. Just as in the baseline model, increasing $b$ has no effect on equilibrium effort $e_{r}^{1*}$ and results in a first-order dominance shift of the steady state distribution of factions. In addition, increasing $b$ has no effect on $e_{r}^{2*}$.

4. Perturbing member $\hat{r}$’s spoils function $\sigma_{\hat{r}}(\cdot)$ affects the equilibrium efforts of faction member $\hat{r}$ but not those of other faction members.

5. As the function $\sigma'_{r}(\cdot)$ shifts upward type-2 effort increases, and type-1 effort decreases or stays constant if it is zero.

6. Suppose we increase the importance of spoils for any member $r$, so we go from $\sigma_{r}(\cdot)$ to $\tilde{\sigma}_{r}(\cdot)$ where $\sigma'_{r}(\cdot) \leq \tilde{\sigma}'_{r}(\cdot)$. Then the steady state distribution of faction size decreases in the sense of first order stochastic dominance, and it does so strictly unless the type-1 effort before the change was equal to zero.

Proof. See Appendix. ■
The last part of Proposition 5 allows us to see the effect of a global adjustment of a faction reward scheme toward weaker team incentives at all ranks $r$. It suffices to adjust the reward scheme for each rank individually, and apply the result iteratively.

Proposition 5 thus shows the benefit to the party of strengthening factional incentives. The stronger are the relative incentives for team effort, the larger are party factions and the greater their contributions of effort to the election of party candidates. In this way, the party does better in elections when it promotes factional ties.

This does not imply that factions are the optimal incentive scheme. If promotions could be conditioned explicitly on team effort, for example, then the party could achieve better results. Some element of factional behavior, however, must be part of the optimal incentive scheme, in the sense that even party members who are not personally running for election need to be induced to exert effort on behalf of the party.

### 6.5 Power Throughout the Hierarchy

We have assumed throughout that the effort of members of higher rank has (weakly) less effect on the provision of public projects. In this section we show that this is merely a technical assumption and explain how to do without it.

The assumption we want to relax is that $\alpha < 1$ in expression (1). The role of that assumption is to ensure that the expression sums to less than 1, and thus can be interpreted as a probability. This assumption can be avoided, at the cost of some slight complication. To see how, suppose we replace expression (1) with the following expression:
\[
\Pr (g = 1) = \min \left\{ 1, \frac{1}{T \cdot (\alpha)} \sum_{r=0}^{T} (\alpha)^r e_r \right\},
\]

where \( \alpha > 1 \) and \( T \) is a positive integer of our choice. This expression meets our desideratum: the effort exerted by officers of a higher rank counts for more.

This expression is almost as tractable as expression (1) because it is linear in the \( e_r \) when it is below 1. And, for factions with less than \( T \) members, expression (9) is definitely smaller than 1 (remember that, in a faction, \( e_r < 1 \)). So the behavior of members of factions with rank smaller than \( T \) is very similar to that described in our model: larger factions exert more aggregate effort and are more likely to survive. Notice that \( T \) can be chosen very large, so that for all of the factions most of the time all our analysis goes through. When the size of the faction exceeds \( T \) it is possible that \( \Pr (g = 1) = 1 \). In that case many combinations of effort among faction members can be an equilibrium, and then the analysis becomes somewhat more cumbersome. But, from the point of view of the evolution of factions, it is not that difficult: factions that exceed a certain number of members get the public project for sure, and so they survive with probability \( b + \Delta \). Thus, nothing substantial in our analysis hinges on the assumption that \( \alpha < 1 \).

### 6.6 Global Public Goods

If a party is forced by its factions to distribute many local public goods to influence local elections, fewer resources may remain to woo voters with promises of public goods in nationwide elections. Thus, we expect parties with strong factions to promise fewer global public
goods; their appeal will be based mostly on their ability to procure local public goods.

To see this point clearly, denote by $E$ the total amount of effort put forth by all the party’s factions. Imagine that a fraction $v$ of this effort results in local public expenditure that could otherwise go toward promising global public goods in national elections (appropriations from a federal budget, say), with the balance $(1 - v)$ representing resources that could not be used for that purpose (the creation of patronage posts in a local hospital, say). Consider the problem of a party president running for nation-wide office. Let us assume that the party president balances the goals of winning local elections with that of winning national office. From the party president’s point of view, increasing the portion $(1 - v)E$ of resources is unambiguously good: these resources help win local elections and do not interfere with nationwide elections. The portion $vE$, on the other hand, has the disadvantage of limiting the party president’s ability to promise global public goods in the national election. If we denote by $Z$ the size of the federal budget, only

$$\max\{0, Z - vE\}$$

is available for the party’s president to promise global public good. When the median voter’s ideal point in the nation-wide election exceeds that level, the party president is constrained in his ability to promise enough global public good. When factions are strong, $E$ is large and the president is more constrained.

This argument does not necessarily favor parties with weak factions: when $v$ is small, the incentive effect of factions dominates over the flexibility-reducing effect, and factions play a useful role on balance. Nevertheless, to the extent that electing the party president in the
nationwide election is a public good for party officers, this argument indicates that factions exert a negative externality on each other. This is analogous to a common-pool problem, where the common pool of resources is $Z$, the size of the federal budget.

### 6.7 Multiple Factional Parties

Up to now we have focussed on the internal workings of one party (call it party 1) and treated its opposition (party 2), asymmetrically, in reduced form. It is, however, straightforward to adjust the model to allow a symmetric treatment of two (or more) competing factional parties. The adjustment merely requires altering the probability that a party wins an election when it is out of office.

Recall that, in the equilibrium of our baseline model, party 1’s probability of winning is increasing in the length of its spell of continuous incumbency in state $s$ (see Propositions 1, 3). In a symmetric, multi-party model, this must be true for any party $h$ which is continuously in power in state $s$. During such a spell, the probability that party $k \neq h$ wins must therefore decrease (as the probabilities of winning must sum to 1). Our current assumptions do not allow this. In our baseline model, when party 1 is out of office the probability that it wins the election in state $s$ is a constant

$$b_s + \Delta_s \cdot e_s^*,$$

(see Section 4), regardless of how many elections it has lost in a row.
Suppose, instead, we amend the probability of winning when out of office to equal

\[ 1 - \left[ b_{2,s} + \Delta_{2,s} (1 - \alpha) \sum_{r=0}^{R_2} \alpha^r c_r^* \right], \]

where the term in brackets captures the probability that party 2 gets re-elected. The term in brackets resembles expression (2), except that here the voter variables are indexed by 2, to allow for the possibility that voters will treat party 2 different from party 1 when in office. Here \( R_2 \) represents the size of party 2’s state-\( s \) faction, which also equals the length of that party’s spell of continuous incumbency. A key feature of this expression is that party 1’s probability of being elected when out of office declines as \( R_2 \) increases, that is, as party 1’s out-of-office spell increases. Note that this probability can never be zero if we assume that \( b_{2,s} + \Delta_{2,s} < 1 \).

The preceding discussion shows that our analysis can be applied to “any party in office” provided we amend the probability of returning to office for the party which out of power. Note that this modification does not affect the parameters of the problem for the incumbent party in state \( s \). Since in our model the public good is generated by the incumbent party, the effects described in Proposition 3 should be maintained. Therefore, we expect our conclusions regarding resource allocations (Section 5) not to change. An extension to the case in which more than two parties compete for office should be similarly straightforward.

### 6.8 Non-linear Effects of Effort on Winning

In our baseline model of factions and in each of the preceding extensions, we assume that the marginal effect of an officer’s effort on the probability of winning is linear and independent
of both his party’s baseline support, $b_s$, and the efforts of his fellow faction members. This assumption precludes the possibility that, upon getting so large or upon gaining control of a very safe district, factions might slack off and deliver fewer public goods. We conclude this section by investigating this possibility with a model that allows a non-linear effect of effort (and baseline support) on winning.

Suppose, in particular, that the rank-$r$ officer solves

$$
\max_{e_r} G \left( b + \Delta \left[ (1 - \alpha) \sum_{r=0}^{R} \alpha^r e_r \right] \right) - c (e_r)
$$

where $G(\cdot)$ is nonlinear, increasing and concave.

If we also assume $c' (0) = 0$ then each member’s problem has an interior maximum which is uniquely identified by the first order condition:

$$
G' \left( b + \Delta \left[ (1 - \alpha) \sum_{j=0}^{R} \alpha^r e_j^{*b,R} \right] \right) \Delta (1 - \alpha) \alpha^r = c' (e_r^{*b,R}) .
$$

Note that in this case the optimal effort $e_r^{*b,R}$ of faction member $r$ will generally depend on the size of the faction $R$, as well as on $b$. Rewrite this as

$$
G' (P_{b,R}) \Delta (1 - \alpha) \alpha^r = c' (e_r^{*b,R}) ,
$$

where

$$
P_{b,R}^* = b + \Delta \left[ (1 - \alpha) \sum_{j=0}^{R} \alpha^r e_j^{*b,R} \right]
$$

represents the probability of faction survival, and

$$(1 - \alpha) \sum_{j=0}^{R} \alpha^r e_j^{*b,R}
$$

is the equilibrium probability of public good provision.
Lemma 1  
1. If $b' > b$ then $P^{*}_{b',R} > P^{*}_{b,R}$, which means that for given faction size the probability of faction survival increases with $b$.

2. If $b' > b$ then for all $r$ we have $e^{s_{b'}R}_{r} < e^{s_{b}R}_{r}$, which means that for given faction size the probability of public good provision decreases with $b$.

3. If $R' > R$ then $P^{*}_{b,R'} > P^{*}_{b,R}$, which means that both the probability of faction survival and the probability of public good provision increase with the size of the faction.

Proof. See Appendix. ■

Parts 1 and 3 of Lemma 1 confirm results of the baseline model: when a locality becomes more of a stronghold ($b$ increases) the probability $P_{b}$ of faction survival goes up; and a larger faction exerts more effort (as a whole), is more likely to survive, and is more likely to deliver local public goods. Parts 1 and 3 together imply that the steady state distribution of faction size shifts (in the sense of stochastic dominance) in the now expected fashion with a change in $b$, and that this shift has the familiar implications on the probability of public good provision. We will call this positive effect on public good provision of an increase in baseline support the distribution effect.

However, part 2 of the Lemma indicates that, for a given faction size, total effort decreases when baseline support increases, and this decreases the probability of public good provision. We will call this negative effect on public good provision of an increase in baseline support the effort effect.

The net impact of an increase in baseline support on the probability of public good provision,
given by
\[ \mathbb{E}_R \left[ (1 - \alpha) \sum_{j=0}^{R} \alpha^r e_j^b \right], \]
thus depends on the relative size of the distribution and effort effects. If \( G \) is closer to linear then the effort effect is dominated by the distribution effect, and a greater stronghold will get more public resources on average. If \( G \) is very concave then the effort effect dominates and a greater stronghold will receive less public resources on average. This observation thus qualifies the extent to which our model accounts for stronghold spending.

It is important to note, however, that the effort effect exists even in a model with only one politician (the standard model). In fact, relative to a model with only one agent (\( R = 1 \)), increasing \( b \) still may improve the long-run probability of public good provision (through its effect on the distribution of \( R \)) whereas in the single-politician model increasing \( b \) with concave \( G(\cdot) \) unambiguously decreases the probability of public good provision. Concavity of \( G(\cdot) \) thus makes increasing \( b \) in the presence of factions less beneficial for public good provision in absolute terms, but increasing \( b \) is more beneficial with factions than in the no-faction world. In this sense, the thrust of our basic model is preserved.

We also note that, while the effect of increasing \( b \) on public good provision is ambiguous in this extended model, the party is unambiguously better off for the presence of factions; factional incentives continue to induces more effort (see Section 6.4), regardless of \( b \). This means that the party organization continues to benefit from encouraging factions.
We presented a new model of factional political competition, where the allocation of resources is driven by coordinated intra-party effort. Despite its simplicity, the model delivers a rich set of implications, both static and dynamic, about the allocation of local and global public goods.

A distinctive feature of the factional model is that the actual power to procure public goods is not vested in elected office: instead, that power is dispersed broadly across the party. When power is not dispersed, our model is equivalent to existing models of distributive politics. In contrast, allowing for the possibility of dispersed power takes us in new directions in the study of political resource allocation, where we need to understand the incentives for, and consequences of, coordinated effort. We suggest that, due to intra-party career incentives, those who hold power coalesce into party factions, which then become the sources of actual, as opposed to formal, power to procure public resources.

The allocation of resources in a factional equilibrium is different from that obtained in other models of distributive politics. Of particular note is the factional model’s predicted bias in favor of party strongholds, a bias which has been reported in many empirical studies of cross-sectional resource allocations. We do not claim that factions are the only source of a stronghold premium. Rather, our point is that factions will accentuate that bias, and generally distort the allocation of resources above and beyond the formal rules of the party and the political system.

We view this paper as a first cut at modeling the incentivization and coordination of intra-
party effort. A central premise of this paper is that, much like workers within a firm, party officers need to be motivated to exert cooperative effort. If this is right, then to understand the resource allocation it becomes crucial to study factions and the intra-party career incentives that generate them. Just as personnel economics has delivered important new insights by looking at intra-firm incentive schemes, we expect that the rules of party organization, and the careers of individual political personnel, will prove to be important factors in predicting the allocation of public resources. We hope much work will be done along those lines.

Given the scope of such an agenda, this paper is not, and is not meant to be, the last word on the subject. To the extent that this paper is successful in arguing for the importance of party factions for the allocation of public resources, the paper raises several questions. If parties truly are aggregations of factions, as we make them out to be, then what are the boundaries of parties? And, indeed, what are the boundaries of factions? How does ideology affect the incentives to form and stick to factions? Questions such as these are important, but they are beyond the scope of this paper. The goal of this paper is to make the case that, when the power to procure public resources is distributed within the party, power networks such as factions become important determinants of the allocation of public resources. To the extent that successfully making this case raises more questions, we view this as pointing a way forward.
Appendix

A Proofs

A.1 Competition among large number of factions

We model the within-party competition for resources as a tournament among a large number of factions. $N$ factions compete for $qN$ prizes (units of the public good) where $q < 1$. We allow a faction to be as small as a single member. Whether a faction $i$ receives the public good depends on both effort and luck; specifically, the $qN$ factions with the greatest influence $r_i$ receive the public good, where

$$ r_i = u_i + (1 - \alpha) \sum_{r=0}^{S} (\alpha)^r e_r. $$

Here $u_i$ is the luck element, the realization from a uniform distribution $U$ with support $[-1, +1]$. The effort element is represented by the discounted sum of the $e_r$. The element $e_r \geq 0$ represents the effort put in by state-$i$ faction member $r$. $S$ is the size of the faction at time $t$.

Faction $i$ wins the public good if and only if $r_i$ exceeds the $q$-th quantile of the empirical distribution of the equilibrium $r$’s. The $q$-th quantile is a random variable. However, since the realizations $u_i$ are uncorrelated across factions, as $N \to \infty$ this quantile converges in probability to a number which we denote by $I_q$. In the limit when the number of factions
grows, faction \(i\) wins a public good if and only if \(r_i \geq I\). Now,

\[
\Pr (r_i \geq I) = \Pr \left( U \geq I - (1 - \alpha) \sum_{r=0}^{S} (\alpha)^r e_r \right) \\
= \frac{3}{2} - \frac{I - (1 - \alpha) \sum_{r=0}^{S} (\alpha)^r e_r}{2}.
\]

To be exact, the second equality holds only when the numerator of the fraction is within the support of \(U\), which must necessarily be the case in equilibrium since no faction would want to exert more effort than it takes to win for sure. Then, in equilibrium the probability that the public good is provided to state \(i\) is given by

\[
\Pr (g_t = 1) = i + \frac{1 - \alpha}{2} \sum_{r=0}^{S} (\alpha)^r e_r, \quad (A1)
\]

where \(i = \frac{3 - I}{2}\). We see that a large faction finds it easier to provide the public good to its constituents.

### A.2 Proof of Proposition 4

**Proof.** We start with a preliminary observation. If equilibrium efforts are symmetric, equation (8) can be split into the following two equations:

\[
\left( \frac{1}{p} - \beta \right) V (n) = [b + \Delta n e^*] [1 + \beta (V (n + 1) - V (n))] - c (e^*) \quad (A2)
\]

\[
c' (e^*) = \Delta [1 + \beta (V (n + 1) - V (n))] \quad (A3)
\]

If \(V (\cdot)\) and \(e^*\) solve the system (A2),(A3) then \(V (\cdot)\) solves equation (8). We now prove each statement in the proposition.
1. If \( V(n) \) has the hypothesized form, then (A3) is satisfied if

\[
e^* = [c']^{-1}(\Delta [1 + \beta K_2])
\]  

(A4)

Now given this \( e^* \) we need to verify that a \( V(n) \) with the hypothesized form solves (A2). To see this substitute into (A2) to get

\[
\left( \frac{1}{p} - \beta \right) \left[ K_1 + K_2 n \right] = \left[ b + \Delta n [c']^{-1}(\Delta [1 + \beta K_2]) \right] [1 + \beta K_2] - c \left( [c']^{-1}(\Delta [1 + \beta K_2]) \right)
\]

Let’s isolate the terms involving \( n \) on both sides.

\[
\left( \frac{1}{p} - \beta \right) K_1 + n K_2 \left( \frac{1}{p} - \beta \right) = n \left\{ \Delta [1 + \beta K_2] \cdot [c']^{-1}(\Delta [1 + \beta K_2]) \right\} + b [1 + \beta K_2] - c \left( [c']^{-1}(\Delta [1 + \beta K_2]) \right)
\]

For this equation to hold for all \( n \) we need the terms involving \( n \) to be equal, which requires

\[
\frac{K_2}{[1 + \beta K_2]} \left( \frac{1}{p} - \beta \right) = \Delta \cdot [c']^{-1}(\Delta [1 + \beta K_2]),
\]

(A5)

or equivalently

\[
c' \left( \frac{K_2}{[1 + \beta K_2]} \left( \frac{1}{p} - \beta \right) \frac{1}{\Delta} \right) = \Delta [1 + \beta K_2].
\]

We want this equation to have a solution. As \( K_2 \to 0 \) the LHS converges to \( c'(0) = 0 \). As \( K_2 \to \infty \) the LHS converges to \( c' \left( \left( \frac{1}{p\beta} - 1 \right) \frac{1}{\Delta} \right) \), so if we assume that \( c'(X) = \infty \) for some \( X < \left( \frac{1}{p\beta} - 1 \right) \frac{1}{\Delta} \) then it follows that the LHS goes to infinity before the RHS.

So the two must cross in the middle. Hence there exists a \( K_2^* \) that solves (A5).

If (A5) has a solution then equation (A2) reduces to

\[
\left( \frac{1}{p} - \beta \right) K_1 = b [1 + \beta K_2^*] - c \left( [c']^{-1}(\Delta [1 + \beta K_2^*]) \right).
\]
For any given $K_2^*$ there exists a unique $K_1^*$ which solves this equation, and hence (A2).

Thus we have shown that, under assumption 6.2, there is an equilibrium to dynamic game in which the value function is linear affine in $n$.

2. First observe that $K_2^*$ is independent of $n$ and of $b$, since it is defined as the solution to equation (A5) which does not involve $n$ or $b$. Now, from (A4) we have $e^* = [e']^{-1} (\Delta [1 + \beta K_2^*])$, which is independent of $n$ and $b$.

3. Since $e^*$ is independent of $b$ and $n$, it follows that the probability of being elected

$$\Pr (E|n, e^*) = b + \Delta ne^*$$

is increasing in $b$ and $n$.

4. Let $\pi_t (R)$ denote the probability that a faction has size $R$ at time $t$. Then $\pi_{t+1} (R)$ is equal to the probability that the faction previously of size $R$ lost its election, plus the probability that a faction previously of size $R - 1$ won its election, both multiplied by the probability that either faction survived. Formally, for $R > 0$ we have

$$\pi_{t+1} (R) = p \{ \pi_{t+1} (R) [1 - \Pr (E|R, e^*)] + \pi_t (R - 1) \Pr (E|R - 1, e^*) \}.$$

At the ergodic distribution the time subscripts disappear and so we have

$$\pi (R) = p \{ \pi (R) [1 - \Pr (E|R, e^*)] + \pi (R - 1) \Pr (E|R - 1, e^*) \}.$$

Rearranging we get

$$\frac{\pi (R)}{p} - \pi (R) [1 - \Pr (E|R, e^*)] = \pi (R - 1) \Pr (E|R - 1, e^*).$$
\[
\frac{\pi(R)}{\pi(R-1)} \left( \frac{1-p}{p} + \Pr(E|R, e^*) \right) = \Pr(E|R-1, e^*). 
\]

and so, finally,
\[
\frac{\pi(R)}{\pi(R-1)} = \frac{\Pr(E|R-1, e^*)}{\left( \frac{1-p}{p} + \Pr(E|R, e^*) \right)}. 
\]

We know that \( \pi(0) > 0 \) due to our assumption that \( p > 0 \). Thus the denominator in the LHS of this equation is nonzero, so the equation is well defined. Since \( \Pr(E|R-1, e^*) < \Pr(E|R, e^*) \), we have \( \pi(R-1) > \pi(R) \), that is, the ergodic distribution of faction size is decreasing in size.

5. As \( b \) increases the ratio
\[
\frac{\Pr(E|R-1, e^*)}{\left( \frac{1-p}{p} + \Pr(E|R, e^*) \right)} \] \( = \frac{b + (R-1) \Delta e_r^*}{\frac{1-p}{p} + b + R \Delta e_r^*} \)

increases towards 1, which means that the distribution of faction size becomes flatter. Hence increasing \( b \) produces a stochastic dominance shift in the distribution of faction sizes. Since the probability of the public good being provided is an increasing function of faction size, localities with a higher \( b \) will have a higher average probability of receiving the public good (using stochastic dominance).

\[
\square 
\]

A.3 Proof of Proposition 5

Proof.
1. Concavity of \( \sigma \) implies that the faction member’s maximization problem is concave in \( e_{r*}, e_{r} \). So there exists a unique maximum. Moreover: neither \( e_{r*}^{1} \) nor \( e_{r*}^{2} \) can equal 1, since then the cost of effort would be infinite; and since \( \sigma'(0) = \infty \) then \( e_{r*}^{2} \) must exceed zero. Therefore the only possible non-interiority of the solution is the case \( e_{r*}^{1} = 0 \).

For future reference we note that if \( e_{r*}^{1} > 0 \) then the solution solves the system

\[
\Delta (1 - \alpha) \alpha^r - c' (e_{r*}^{1} + e_{r*}^{2}) = 0 \tag{A6}
\]

\[
\sigma'_r (e_{r*}^{2}) - c' (e_{r*}^{1} + e_{r*}^{2}) = 0.
\]

If \( e_{r*}^{1} = 0 \) then the solution solves the system

\[
\Delta (1 - \alpha) \alpha^r - c' (e_{r*}^{2}) \leq 0 \tag{A7}
\]

\[
\sigma'_r (e_{r*}^{2}) - c' (e_{r*}^{2}) = 0.
\]

The two systems of equations (A6) and (A7) represent the usual necessary conditions for local optimality in the interior and non-interior case, respectively. The set of primitives under which the interior or non-interior maximum prevail are mutually exclusive. This is due to the concavity of the maximand, which guarantees a unique local (and global) maximum. Thus if (A6) has a solution then there is an interior global maximum and no extreme local maxima, so (A7) cannot have a solution. Viceversa, if (A7) admits a solution then (A6) cannot have one. Thus we can conveniently check whether a constellation of primitives gives rise to an interior or non-interior maximum simply by checking whether (A7) has a solution. We will make repeated use of this observation.
2. Now we show that if \( \sigma_r(\cdot) = \sigma(\cdot) \) independent of \( r \) then type-1 equilibrium effort is zero for \( r \) large enough. To see this observe that the second equation in (A7) always has exactly one solution, and that this solution will solve the first equation for \( r \) large enough.

Now we show that type-1 effort is decreasing in \( r \) if \( \sigma_r(\cdot) = \sigma(\cdot) \) independent of \( r \).

The proof is in two parts. First, we observe that, all other things equal, it is easier to satisfy (A7) if \( r \) is large. This means that the set of function pairs \( \{c(\cdot), \sigma(\cdot)\} \) such that (A7) has a solution is nondecreasing in \( r \), which means that if \( e_1^{1*} = 0 \) then \( e_1^{1*} = 0 \) for all \( r' > r \). To finish the argument it suffices to show that if \( e_1^{1*} > 0 \) then \( e_1^{1*} > e_1^{1*} \) for all \( r' < r \). To see this, observe first that since it is harder to satisfy (A7) for \( r' \) than it is for \( r \), and since A7 is violated for \( r \) (because \( e_1^{1*} > 0 \) ) then A7 is violated for \( r' \) too (so \( e_1^{1*} > 0 \)). This means that both \( e_1^{1*} \) and \( e_1^{1*} \) solve system (A6). From (A6) we have

\[
\Delta (1 - \alpha) \alpha^r = \sigma'(e_2^{2*}),
\]

which since \( \sigma'(\cdot) \) is decreasing implies that

\[
e_2^{2*} > e_2^{2*}.
\]

At the same time, (A6) requires

\[
\Delta (1 - \alpha) \alpha^r = c'(e_1^{1*} + e_2^{2*})
\]

which since \( c'(\cdot) \) is increasing implies \( e_1^{1*} + e_2^{2*} < e_1^{1*} + e_2^{2*} \). Rewriting this inequality yields

\[
e_1^{1*} - e_1^{1*} < e_2^{2*} - e_2^{2*} < 0,
\]
where the last inequality follows from (A8). Therefore \( c_r^{1*} < c_r^{1*} \), which proves that type-1 equilibrium effort is decreasing in rank.

3. The proof follows trivially from the observation that equations (A6) and (A7) are independent of \( \beta \).

4. To see this, observe that the equations (A6) and (A7) for member \( r \) do not involve \( \sigma_r \), nor do they involve the effort of any faction member different from \( r \). Therefore perturbing member \( r \)'s spoils function \( \sigma_r (\cdot) \) has neither direct nor indirect effects on the equilibrium efforts of member \( r \).

5. An upward shift of the function \( \sigma_r' (\cdot) \) to \( \tilde{\sigma}_r' (\cdot) \) makes it easier, all other things equal, to satisfy (A7). This means that the set of functions \( c (\cdot) \) such that (A7) has a solution is bigger under \( \tilde{\sigma}_r' (\cdot) \), which means that if \( e_r^{1*} = 0 \) then \( e_r^{1*} = 0 \).

To finish the argument it suffices to show that if \( e_r^{1*} > 0 \) then \( e_r^{1*} > e_r^{1*} \). To see this, observe first that since it is harder to satisfy (A7) under \( \sigma_r' (\cdot) \) than under \( \tilde{\sigma}_r' (\cdot) \), and since A7 is violated for \( \sigma_r' (\cdot) \) (because \( e_r^{1*} > 0 \)) then A7 is violated for \( \sigma_r' (\cdot) \) too (so \( e_r^{1*} > 0 \)). This means that the solution is interior in both cases. It then follows from (A6) that \( e_r^{2*} \) and \( \tilde{e}_r^{2*} \) solve

\[
\tilde{\sigma}_r' (\tilde{e}_r^{2*}) = \Delta (1 - \alpha) \alpha^r = \sigma_r' (e_r^{2*}) .
\]

Since \( \sigma_r' (\cdot) < \tilde{\sigma}_r' (\cdot) \) and both functions are decreasing in their arguments, it follows that \( \tilde{e}_r^{2*} > e_r^{2*} \). Moreover, from (A6) we have

\[
c' (\tilde{e}_r^{1*} + e_r^{2*}) = \Delta (1 - \alpha) \alpha^r = c' (e_r^{1*} + e_r^{2*}) .
\]
which means that the sum of equilibrium efforts is a constant independent of \( \sigma'_r(\cdot) \).

Since \( \bar{e}_r^{2*} > e_r^{2*} \), it follows that \( \bar{e}_r^{1*} < e_r^{1*} \).

6. To see this, observe first that based on the previous items we know that all type-1 efforts are unchanged except \( e_r^{1*} \) which, if strictly positive, decreases to \( \bar{e}_r^{1*} \). So if we denote by \( \pi \) the steady state distribution before the increase, and by \( \bar{\pi} \) the one after the increase, we can use equation (4) to conclude that

\[
\frac{\pi (R + 1)}{\pi (R)} = \frac{\bar{\pi} (R + 1)}{\bar{\pi} (R)} \quad \text{for } R < r \\
\frac{\pi (R + 1)}{\pi (R)} > \frac{\bar{\pi} (R + 1)}{\bar{\pi} (R)} \quad \text{for } R \geq r.
\]

The first equation implies that the functions \( \pi (R) \) and \( \bar{\pi} (R) \) never cross for for \( R < r \)—unless they coincide for all those \( R \)'s, which as we will see is not possible. The second equation implies that if \( \pi (R) \) and \( \bar{\pi} (R) \) cross at some \( R^C \geq r \) then they only cross once, and \( \pi (R) > \bar{\pi} (R) \) if \( R > R^C \). From the observation that the two functions must cross for otherwise they cannot both sum to 1, it follows that there exists a value \( R^C \) such that \( \pi (R) > \bar{\pi} (R) \) if \( R > R^C \), and \( \pi (R) < \bar{\pi} (R) \) if \( R < R^C \). Therefore \( \pi (R) \) stochastically dominates \( \bar{\pi} (R) \).

\[\blacksquare\]

A.4 Proof of Lemma 1

Proof.
1. Suppose not. Then it must be $\sum_{r=0}^{R} \alpha^r e_r^{sb'} < \sum_{r=0}^{R} \alpha^r e_r^{sb}$ and so there is at least one $r$ such that

$$e_r^{sb'} < e_r^{sb}. \quad (A9)$$

For that $r$ we have

$$G' (P_{b,R}) \Delta (1 - \alpha) \alpha^r = c' (e_r^{sb}) \quad (A10)$$

$$G' (P_{b',R}) \Delta (1 - \alpha) \alpha^r = c' (e_r^{sb'}) \quad (A11)$$

By assumption $P_{b',R} \leq P_{b,R}$ and $G' (\cdot)$ is a decreasing function, so $G' (P_{b',R}) \geq G' (P_{b,R})$.

Then (A10) and (A11) imply $c' (e_r^{sb'}) \geq c' (e_r^{sb})$ and so, by convexity of $c' (\cdot)$,

$$e_r^{sb'} \geq e_r^{sb}.$$

This inequality contradicts (A9) and, therefore, proves our claim.

2. If $b' > b$ then we know $P_{b'} > P_b$ and then (A10) and (A11) imply $c' (e_r^{sb'}) < c' (e_r^{sb})$ and so, by convexity of $c' (\cdot)$,

$$e_r^{sb'} < e_r^{sb}$$

for all $r$.

3. Suppose not. Then it must be $\sum_{r=0}^{R'} \alpha^r e_r^{sb,R'} < \sum_{r=0}^{R} \alpha^r e_r^{sb,R}$ and so there is at least one $r$ such that

$$e_r^{sb,R'} < e_r^{sb,R}. \quad (A12)$$

For that $r$ we have

$$G' (P_{b,R}) \Delta (1 - \alpha) \alpha^r = c' (e_r^{sb,R}) \quad (A13)$$

$$G' (P_{b,R}) \Delta (1 - \alpha) \alpha^r = c' (e_r^{sb,R}). \quad (A14)$$
By assumption $P_{b,R'} \leq P_{b,R}$ and $G' (\cdot)$ is a decreasing function, so $G' (P_{b,R'}) \geq G' (P_{b,R})$.

Then (A13) and (A14) imply $c' (e_{r}^{b,R'}) \geq c' (e_{r}^{b,R})$ and so, by convexity of $c' (\cdot)$,

$$e_{r}^{sb,R'} \geq e_{r}^{sb,R}.$$ 

This inequality contradicts (A12) and, therefore, proves our claim.
References


Figure Legends

Figure 1: Evolution of factions between periods.

Figure 2: Steady-state distribution $\pi$ of faction size in state $s$. 
<table>
<thead>
<tr>
<th>Study</th>
<th>Form of Spending</th>
<th>Country</th>
<th>Years</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khemani (2007)</td>
<td>Fed. transfers to states</td>
<td>India</td>
<td>1972-1995</td>
<td>both</td>
</tr>
<tr>
<td>Arulampalam et al. (2008)</td>
<td>Fed. grants to states</td>
<td>India</td>
<td>1974-1997</td>
<td>both</td>
</tr>
</tbody>
</table>

Table 1: Selection of empirical studies that find spending favors party strongholds, or no evidence that spending favors swing districts. Findings are coded as significant evidence of stronghold district spending (stronghold), swing district spending, neither or both.