

Committee Design with Endogenous Information

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Identical agents gather costly information, and then aggregate it through voting. Because information is a public good, information is underprovided relative to the social optimum. A “good” voting rule must give incentives to acquire information, as well as aggregate information efficiently. A voting rule that requires a large plurality (in the extreme, unanimity) to upset the status quo can be optimal only if the information available to each agent is sufficiently accurate. This result is independent of the preferences of voters and of the cost of information.

1. INTRODUCTION

This paper is concerned with the arrangements for collective decision making in a world where agents must be motivated to acquire information. The analysis identifies some basic forces shaping the design of panels of decision makers—referees, managers, jurors, etc.—in situations where decision makers are useful only insofar as they expend some effort to gain information about the alternatives at hand. These situations are common, one being the refereeing process, where an editor requires the opinions of a number of experts who must read the paper (acquire information) in order to give an opinion as to publication. Analogously, in committees which screen applicants to a programme, the committee members must gain information about the applicant’s qualifications and likelihood of success in order to evaluate whether to admit him/her. Finally, in trial juries it is important that jurors pay attention to the evidence in order to make an informed judgement.¹

We focus on environments where information is a public good in the sense that the social benefits of one decision maker acquiring information exceed the private benefits. A good mechanism in this environment must not only aggregate information efficiently, but also induce the decision makers to acquire information. We analyse the design of voting mechanisms. Our problem is to choose two parameters which determine both the incentive to acquire information and the efficiency with which information is aggregated. The first parameter is the size of the decision-making unit: the number of committee members. The second is the plurality required to overturn the status quo: the voting rule.

Our main result is this: a voting rule that requires a large plurality (in the extreme, unanimity) to upset the status quo can be optimal only if the information available to each committee member is sufficiently accurate. When individual information is noisy, the drawback of requiring a large measure of consensus to switch from a status quo is that doing so attenuates the committee members’ incentives to invest in information. To see why, take the most

1. The importance of the jurors’ attentiveness is reflected in the concern within the judicial system with the attentiveness of alternate jurors, those “spare” jurors who are called to vote only in case of inability of a member of the original jury. See the Report by the Special Committee of the New York State Bar Association (1999) for a discussion of this problem and recommendations.

consensual voting rule, unanimity. The only circumstance in which a committee member has any use for his information is when the remaining $n - 1$ members vote to switch from the status quo. But this is an unlikely event since, if individual information is noisy, committee members are likely to have different perceptions about the best policy. If information is costly and must be acquired before the voting stage, committee members will not acquire it. Thus, the drawback of consensual decision making is that committee members may not invest in information because they expect that their vote will not influence the final decision.

The standard model of collective decisions under uncertainty does not speak to this issue because it assumes that decision makers are endowed with (costless) information. In the most basic version (Young (1988), Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997, 1998)), there are n agents with identical preferences. There are two alternatives, a status quo and an alternative policy. Agents prefer the status quo in state of the world 1, but not in state of the world 2. Agents are endowed with noisy binary signals about the state of the world, and aggregate their information through voting.

In this world, any voting rule can be optimal, depending on the agents' preferences. Characterize a voting rule by R , the minimum number of votes required to switch from the status quo. If agents are very concerned with mistakenly abandoning the status quo, the optimal value of R/n will be high. If instead their main concern is to be stuck with the status quo when the alternative policy is preferable, R/n will be low.² In contrast, in a world where information is endogenous, we show that the optimal value of R/n can be bounded independently of the agents' preferences.

Moreover, the model with exogenous information yields implausible normative results on the question of committee size. In that model, increasing the number of committee members unambiguously helps the decision process, as each member contributes his private information. For instance, in the face of public concern about the unpredictability of jury verdicts,³ the existing model prescribes a (presumably large) increase in the size of juries. Why are committees not expanded in size, if that increases the accuracy of their decision? Common sense suggests that if committees and juries were enlarged, committee members would become less responsible for the decision, and thereby less engaged in the collection of information. In fact, this argument is usually invoked to explain why the average voter is so poorly informed in large elections.

This points to a model where information is endogenous, *i.e.* acquired by committee members. In such a model, increasing the size of the committee need not increase the aggregate information of its members, because a member's incentives to acquire information decrease with committee size. This feature captures the previous argument for keeping committees small. For this model, we derive an interesting characterization of the optimal voting rule.

We consider the same environment as in the standard model described previously, except for the information structure. We analyse a game where: first, n agents are selected to become committee members; at this stage, agents have no private information about which of the two alternatives is better. Then, each committee members decides whether to incur a cost to learn a noisy signal about the state of the world. The signal is binary, and is accurate—*i.e.* reveals the true state—with probability p . Finally, the committee members participate in a voting game. The rules of the game are that the status quo is implemented unless R or more members vote "switch".

2. To determine the optimal voting rule, consider a statistician who has the same utility function as the decision makers, and observes their signals directly. The optimal decision rule is characterized by an integer R ; the statistician implements the alternative policy if at least R signals indicate state of the world 2. It is clear that the analogous voting rule, which switches from the status quo if at least R agents vote "switch", is the optimal voting rule. Under this voting rule, it is a Nash equilibrium for agents to vote sincerely, *i.e.* to vote "switch" if and only if their signal indicates state 2. This is also the best possible outcome given the information in the economy, because it implements the decision of a benevolent statistician/social planner.

3. Concerning public mistrust of the accuracy of jury decisions see "Jury Trials: A Mystery", *The Economist*, January 27, 1996. The U.K. recently moved to eliminate jury trials in cases that deal with several middle-ranking crimes.

A mechanism designer, with preferences identical to the agents', picks R and n to maximize his expected utility from the decision.

We show that, regardless of preferences and cost of information, the optimal value of R/n can never be greater than approximately p , the accuracy of the signal. This statement is interesting because it does not depend on the agents' preferences or on the cost of information. Instead, the bound on R/n depends on the quality of information available to agents. As a consequence, a rule in which a high level of consensus is necessary to change the status quo (in the extreme, unanimity) can be optimal only for high values of p . In particular, our analysis shows that consensual voting rules cannot be justified purely based on the cost of the mistake; whatever the relative cost of mistakes, consensual rules cannot be optimal unless the available information is sufficiently accurate.

Consider the implications for jury design. In contrast to the model with a finite number of exogenously informed jurors, in which unanimity can be the optimal rule if the jurors' preferences are sufficiently averse to convicting the innocent, in our model preferences alone are not enough to justify unanimity as the optimal rule; it is also necessary that jurors be very confident in their information.

The reason that consensual rules cannot be optimal is that pieces of information are complementary goods when the optimal rule is consensual. We show that when R/n is close to one, an agent's value for information is increasing in the number of informed agents. In view of this, it is profitable to increase the committee size to $n + 2$, while adjusting the voting rule to $R + 1$.⁴

It is surprising that the value of information can be increasing in the number of committee members. To understand this phenomenon, it is crucial to remark that as n increases, we are also increasing R . Now, suppose we start from unanimity rule with n members. As we increase the size of the committee to $n + 2$ and we correspondingly adjust the voting rule, the ratio *Rule/number* moves closer to $1/2$, and hence to p if p is close to $1/2$. This tends to increase the chances that committee members are pivotal, because percentages close to p of signals favourable to the alternative policy are more likely (happen more often) than percentages close to 1. Indeed, it is unlikely that *all* committee members get a signal favourable to the alternative, even if the alternative is better than the status quo; in fact, if the alternative is better, the most likely configuration is to have pn signals in favour of the alternative out of n committee members. Thus, as n increases the ratio *Rule/number* decreases towards p and the incentive to stay informed increases.

Increasing committee size remains unambiguously profitable until the ratio R/n becomes smaller than approximately p . From that point on, the agents' value for information is decreasing in the number of informed agents. This is because now, as the ratio *Rule/number* moves closer to $1/2$ it moves away from p ; therefore, the force discussed above works toward making it less likely that committee members are pivotal. Still, it continues to be profitable to expand the committee size until the incentive to stay informed (which is now decreasing in n) falls below the cost of information. If the cost of information is low, this can take a large number of additional committee members, and in this case the optimal voting rule will be close to simple majority.

It is interesting to determine how the optimal jury size responds to changes in the parameters. Obviously, larger juries are optimal when the cost of information decreases. Under the condition that the cost of information be small, we are able to provide further comparative statics results. Optimal committee size tends to increase as the prior probability that one

4. Increasing R is good for information aggregation since if with n jurors the optimal rule from the information aggregation viewpoint is R , then with $n + 2$ jurors the optimal rule becomes $R + 1$; in addition, we show that it is necessary to maintain the incentives to stay informed.

alternative is better becomes more diffuse, or as the costs of the two types of errors becomes more similar. And optimal committee size tends to decrease as the quality of information p increases.

Section 2 presents the model. Section 3 lays the ground for the analysis of Sections 4 and 5, which contain the main results. In Section 6 we contrast our result on consensual rules with the result of Feddersen and Pesendorfer (1998) concerning unanimity. In Section 7 we relax the assumption that the mechanism designer only controls the size of the committee and the voting rule. We show that if we allow the mechanism designer to make arbitrarily large transfers, the designer can trivially achieve an arbitrarily precise decision.

1.1. *Related literature*

There is a vast early literature on the Condorcet Jury theorem (starting, of course, with Condorcet); good summaries are in Miller (1986) and Grofman and Owen (1986). Young (1988) discusses majority rule as a means of aggregating private information. Austen-Smith and Banks (1996) point out that, in situations such as Young's, voters generally vote strategically even if they have identical preferences. McLennan (1998) argues that strategic voting is not the source of inefficiency in common interest games (such as juries with homogeneous jurors). Feddersen and Pesendorfer (1997) show that large numbers of voters voting strategically aggregate information perfectly under any voting rule R that, roughly, can be expressed as a nonzero fraction of n as n goes to infinity (for example, simple majority has $R = n/2$). Furthermore, Feddersen and Pesendorfer (1998) show that information aggregation cannot happen under unanimity rule; we discuss this result in Section 7. Coughlan (2000) argues that unanimity can be an optimal rule if communication among jurors is allowed and jurors have similar preferences. Guarnaschelli, Palfrey and McKelvey (2000) find that in an experimental setting with homogeneous jurors communication does not happen to the extent predicted by the theory, and Doraszelsky, Gerardi and Squintani (1999) show that when jurors are heterogeneous communication is only partially informative. In the present paper we study the case where communication is not allowed; this assumption is discussed at length in Section 2.1.

Query 1

Chwe (1999) is concerned with minority representation. He considers the mechanism design problem of maximizing the utility of one type of juror (the majority), and gives sufficient conditions under which it is optimal to give some decision power to another type of juror (the minority). The majority wants to delegate some of its decision power if the minority's interests are sufficiently collinear with its own's. This is because the minority is endowed with information that is valuable to the majority. Therefore, when the majority is very uncertain about which alternative is best, it finds it beneficial to include the minority into the decision process.

Query 2

While most of the literature on juries has ignored the issue of information acquisition, there are some recent exceptions. Mukhopadhaya (1998) analyses a jury with endogenous information. He assumes majority rule, and concentrates on symmetric mixed strategy equilibria. He shows that, for certain parameter values, increasing the jury size may lead to a less accurate decision. Li (2001) analyses a setup where a fixed number of jurors each purchase the precision of a noisy signal, which is public information. It is shown that there is an incentive to distort the rule away from the rule that would be optimal if information was exogenously given. Cai (2001) studies a case in which agents acquire information about the state of the world and their own private preferences, and then report their messages to a central planner who takes a decision. The central planner makes the decision without being bound by any rules, so this model is similar to a cheap talk model. In Gerardi and Yariv (2002) jurors can acquire information and communicate with each other. In the case of homogeneous jurors it is shown that communication allows jurors to coordinate on equilibria that are, while inefficient for given number of jurors, nevertheless

give strong incentives to acquire information. Gerardi and Yariv show that sometimes the loss in efficiency is more than compensated by the greater amount of information acquired in equilibrium, relative to the case in which there is no communication. Finally, Borghes (2001) considers a model in which values are private—so information aggregation is not an issue—and it is costly for agents to vote. It is shown that forcing agents to vote is Pareto-dominated by a laissez faire policy.

Query 3

2. THE MODEL

We use the terminology of jury models. There are two states of the world, I and G (Innocent and Guilty). There are two possible outcomes, conviction or acquittal.

There is an infinite supply of identical agents. Agents assign prior probability $P(I)$ and $P(G)$ to the two states of the world. Agents care about the outcome of the process: $-q$ is their cost of convicting the innocent, $-(1 - q)$ is the cost of acquitting the guilty. Taking the right decision gives 0. Agents maximize expected utility.

Each agent can pay a cost c and acquire a signal $X \in \{i, g\}$. The conditional probabilities of signals are $P(X = i | I) = P(X = g | G) = p > 1/2$.⁵ We designate as the *status quo* the decision which would have been taken without any collection of information.

A voting game of n jurors with rule R is a game where each juror announces Convict or Acquit, and if R or more jurors vote to convict, the defendant is convicted. A mechanism designer picks n and R .

We define a *mechanism* as the following game:

- Stage (1) The mechanism designer picks n jurors and sets the voting rule R .
- Stage (2) The jurors observe R and n . Each juror either gets informed, at a cost c , or not.
- Stage (3) Each juror votes, without observing whether other jurors have acquired information in the previous stage. If R or more jurors vote Convict, the defendant is convicted.

The mechanism designer has valuation q and $1 - q$ for types I and II errors, just as agents. He maximizes his expected utility from the decision. In addition, the mechanism designer takes into account a per-juror cost λ that is lexicographic with respect to his expected utility. This means that, when choosing between two juries which give different expected utility (expected cost of mistake), the mechanism designer will always choose the jury which gives the highest expected utility irrespective of the number of jurors. However, when choosing among two juries which deliver the same expected utility, the mechanism designer will choose the one with the smallest number of jurors.

Our setup—as usually the case in voting games—exhibits many equilibria. In this paper we restrict attention to equilibria which have two properties. First, they are in *monotone strategies*, in the sense that for all jurors the probability of voting “convict” is higher if the signal is guilty than if it is innocent. Second, jurors use *pure strategies*, that is, they do not randomize on their vote or on their decision of acquiring information. Of course these restrictions are on equilibria, not on strategies. Combining these restrictions guarantees that in equilibrium the event in which a juror is pivotal exactly pins down the ratio of guilty to innocent signals among other jurors; this feature guarantees monotonicity of the best response to a strategy profile that is monotone and pure. Equilibria that are in monotone pure strategies can display interesting strategic behaviour,

5. Information acquisition could be reinterpreted as information processing. In that case, the cost c captures the effort that each decision maker puts into updating his beliefs given the available information.

as jurors can (and generally will) vote uninformatively by using constant strategies that ignore their information.

We regard the restriction to equilibria in monotone strategies as reasonable, and few papers about voting with incomplete information consider non-monotone equilibria. As a nice by-product of this restriction we get that the equilibria in our simultaneous voting game are also equilibria in a sequential version of the voting game (see Dekel and Piccione, 2000, Theorem 4). Restricting attention to equilibria in pure strategies is a more constraining limitation, which we discuss in the conclusions.

2.1. *Discussion of the model*

We assume that, at the voting stage, jurors cannot share information. This prevents jurors from coordinating at the voting stage, which would allow them to implement the optimal outcome regardless of the voting rule. Let us first discuss the appropriateness of this assumption from the point of view of modelling. While forbidding communication allows for equilibria that are suboptimal in the voting stage (these equilibria would not be robust to communication and renegotiation), we end up showing that optimality in the whole game prescribes optimality at the voting stage. Therefore, the assumption that jurors cannot share information does not stack the deck in favour of our results. In this sense, the fact that we assume away communication makes the results stronger, not weaker; our results would not change if jurors could communicate at the voting stage and renegotiate to the best equilibrium. An additional point is that assuming that jurors cannot communicate seems a reasonable first step towards analysing a more general environment where truthful sharing of information is hampered by the unobservable heterogeneity of jurors (see Doraszelsky *et al.*, 1999 for such a model).⁶ Gerardi and Yariv (2002) separate the issue of communication from that of renegotiation. They show that communication can provide stronger incentives to acquire information and thus improve on the outcomes of the game we study.

In terms of realism, the assumption that communication is not allowed is a stylized description of environments where communicating is costly. This description is not far from accurate in a number of practical instances of collective decision-making. One such instance occurs when committee members, or experts, are in different physical locations; this is the case, for example, in the refereeing procedures for articles and proposals, where experts do not interact at all when they deliver their recommendations. Additionally, communication can be costly *per se*, as it takes time to discuss opinions and convey information. As evidence of the cost of communicating, in committees that make many repeated decisions (such as deciding admissions to a large program) discussing each single case is often impractical and so the decision is based on aggregating reports prepared privately by each committee member.

If committee members could costlessly and perfectly communicate their information, the voting rule would simply be an immaterial veil that members see through. Coughlan (2000) presents a model where jurors with collinear preferences share their information before voting and the voting rule does not matter. Our assumption that information cannot be shared puts us in the case which is the polar opposite. We view our approach as complementary, not alternative,

6. In a previous version of this paper a stylized model was studied in which the preferences of each juror are his private information. The analysis of the model with identical jurors has a close counterpart in that framework. We view this as an indication that an analysis of the case of homogeneous jurors can be a building block towards more complex models in which communication is impeded by unobservable heterogeneity. Nevertheless, it is important to recognize that the optimal rule in a heterogeneous committee will differ from the rule in a homogeneous committee, due to the jurors' incentives to misrepresent their private information. These incentives, and the resulting distortion in the optimal rule, are more important the greater the heterogeneity. Therefore, the properties of the optimal rule presented in this paper most directly extend to those committees in which heterogeneity is not too large.

to Coughlan (2000) since in most real-world settings the effectiveness of communication is probably somewhere in between the two extremes. For instance, in an experimental setting with pure common values, Guarnaschelli *et al.* (2000) report that, after conditioning on the outcome of a public pre-voting straw poll “the individuals’ private information has a significant effect on the final vote for all combinations of treatment variables”, suggesting that even in highly controlled experimental settings information is not perfectly shared.⁷ In the same vein, Feddersen and Pesendorfer (1998, p. 31) discuss empirical evidence that information is imperfectly shared in trial juries. Consistent with their finding of imperfect sharing of information, Guarnaschelli *et al.* (2000) report evidence that, even after a straw poll, the voting rule matters for the outcome,⁸ and Kalven and Zeisel (1996, p. 461) attest to the fact that allowing less than unanimous verdicts significantly impacts the decision of trial juries.

Query 4
Query 5

In sum, we believe that there are sound theoretical reasons to be interested in the specification of the model with no information sharing, that this assumption usefully approximates a number of interesting real-world situations, and that the forces portrayed in our stylized model will carry over to more elaborate model where communication is possible but imperfect or only partially effective.

Another assumption of our model is that the mechanism designer does not take into account the cost of information incurred by jurors. This introduces a difference in objective between the mechanism designer and jurors, in the spirit of principal–agent theory. This modelling choice is appropriate to model decisions which have an impact on a large number of people besides jurors; in that case, the information costs incurred by the jurors are negligible relative to the benefits of an accurate decision. This argument can be made formal by modelling a mechanism designer who maximizes social utility (including the juror’s information cost), and then letting the number of people in society increase. In Section 4 we show that if the mechanism designer cares about the information costs our main result continues to hold.

The only role of the lexicographic cost λ is to lead the mechanism designer to prefer the smallest among all juries that deliver a given expected utility. Thus, we rule out as suboptimal those environments where some jurors do not get informed with probability one. However, the cost λ does not affect the choice among two juries that yield different expected utilities. In particular, λ does not prevent the mechanism designer from increasing the jury size whenever this enhances the accuracy of the decision.

We assume that p , the probability of the signal revealing the true state is the same in each state. This simplifies the algebra, but a result similar to Theorem 1 can be derived for the case where the accuracy of the signal is state-dependent. This result is available from the author upon request.

2.2. The statistical rule $R^S(n)$

Fix the number of jurors at n , and assume that all jurors are informed. We say that a juror votes *sincerely* if his vote replicates his signal.

Definition 1. A juror votes *sincerely* if he votes Convict if and only if he receives $X = g$.

If all jurors vote sincerely, a juror’s expected payoff from a rule R is

$$\Pi(R, n) = -qP(I) \sum_{x \geq R} \binom{n}{x} (1-p)^x p^{n-x} - (1-q)P(G) \sum_{x < R} \binom{n}{x} (1-p)^{n-x} p^x.$$

7. Guarnaschelli *et al.* (2000, p. 414).

8. Guarnaschelli *et al.* (2000, Table 7, p. 415).

Suppose a statistician chooses the voting rule to maximize a juror's expected payoff. If jurors vote sincerely, this reduces to the statistical problem of constructing the most efficient test to discriminate between the alternatives Guilty and Innocent given n independent signals. Under the assumptions of our model, the most efficient test takes the form: "convict if and only if the sum of guilty signals is $R^S(n)$ or more". We call $R^S(n)$ the *statistical rule*.

Definition 2. The rule $R^S(n) \in \arg \max_R \Pi(R, n)$ is called the **statistical rule**.

Next, we characterize an important property of the statistical rule. Irrespective of primitives, $R^S(n)$ grows at half the speed of n .

Lemma 1. *Suppose $R^S(n) \neq 0, n + 1$. Then $R^S(n + 2) = R^S(n) + 1$.*

Proof. The rule $R^S(n)$ maximizes the payoff $\Pi(R, n)$, hence

$$\begin{cases} \Pi(R + 1, n) - \Pi(R, n) \geq 0 & \text{for } R < R^S(n) \\ \Pi(R + 1, n) - \Pi(R, n) \leq 0 & \text{for } R \geq R^S(n) . \end{cases}$$

Compute

$$\begin{aligned} \Pi(R + 1, n) - \Pi(R, n) &= qP(I) \binom{n}{R} (1-p)^R p^{n-R} - (1-q)P(G) \binom{n}{R} (1-p)^{n-R} p^R \\ &= \binom{n}{R} (1-p)^{n-R} p^R \left[qP(I) \left(\frac{1-p}{p} \right)^{2R-n} - (1-q)P(G) \right]. \end{aligned} \tag{1}$$

We are interested in the sign of this expression. Define

$$h(2R - n) = qP(I) \left(\frac{1-p}{p} \right)^{2R-n} - (1-q)P(G).$$

The sign of $\Pi(R + 1, n) - \Pi(R, n)$ is equal to the sign of $h(2R - n)$. Noticing that $h(2(R + 1) - (n + 2)) = h(2R - n)$ yields the result. \parallel

If n is too small, it may be the case that $R^S(n)$ equals 0 or $n + 1$. However, given any specification of $P(I)$ and q , there is a value \underline{n} large enough such that $R^S(\underline{n}) \neq 0, \underline{n} + 1$. For values of n larger than \underline{n} , $R^S(n)$ grows at half the speed of n . One consequence of Lemma 1 is that unanimity can be optimal for at most two jury sizes. If, for example, unanimity is optimal for a jury of size 13, then it cannot be optimal for any jury of size greater than 14. Furthermore, if unanimity is optimal for 13 jurors, then for a jury of size smaller than 12 the rule "always acquit" must be optimal. Another consequence of Lemma 1 is that for large n the ratio $R^S(n)/n$ approaches 1/2; this feature will play an important role in what follows.

3. EQUILIBRIUM WITH n INFORMED JURORS

Throughout this section we fix the number of jurors at n , and assume that all jurors are informed. We solve for the equilibrium of that voting game. In the next sections we discuss information acquisition.

Consider the voting game with n informed jurors and rule $R^S(n)$. If jurors $2 \dots n$ vote sincerely, juror 1 prefers to vote sincerely, because his incentives coincide with the statistician's. Thus, sincere voting is an equilibrium at rule $R^S(n)$. When the rule is different from $R^S(n)$, is

there an equilibrium where everybody votes sincerely? The answer is in the negative, as shown in Austen-Smith and Banks (1996).

The intuition for this result is that, when R is different from $R^S(n)$ the statistician and the jurors want to move R towards $R^S(n)$. Suppose for instance that $R < R^S(n)$, so with sincere voting convictions are more frequent than they optimally should be. Then a juror will try to rectify this by always voting Acquit, regardless of his signal.

Proposition 1 (Austen-Smith and Banks, 1996). *Assume there are n informed jurors. If $R = R^S(n)$ there is an equilibrium in which all jurors votes sincerely. If $R \neq R^S(n)$ there is no equilibrium in which all jurors votes sincerely.*

Proof. See Austen-Smith and Banks (1996). ||

The previous result has interesting implications for a juror’s willingness to reveal his information: if the voting rule differs from the optimal one, a juror prefers not to reveal his information, assuming all other jurors reveal their information. Thus, suboptimal voting rules cause some information to be wasted. To understand how this happens at equilibrium, it is necessary to understand the equilibrium under a suboptimal voting rule.

We wish to describe the most efficient equilibrium in monotone pure strategies.⁹ When $R \neq R^S(n)$, at the most efficient equilibrium a fraction n_1 of the jurors ignore their information and use their vote to correct the voting rule, and the remaining $n - n_1$ jurors vote sincerely. The existence of such equilibria has been noticed by Ladha, Miller and Oppenheimer (2000) and Dekel and Piccione (2000, p. 44), and is related to the analysis of efficient equilibria in McLennan (1998).

Proposition 2. *Assume there are n informed jurors. Then, if $R < R^S(n)$, at the most efficient equilibrium in monotone pure strategies, a number $n_1 \leq n$ of jurors vote Acquit regardless of their signal, and the remaining $n - n_1$ jurors vote sincerely. If $R > R^S(n)$, at the most efficient equilibrium in monotone pure strategies, a number $n_1 \leq n$ of jurors vote Convict regardless of their signal, and the remaining $n - n_1$ jurors vote sincerely.*

Outline of proof. Here we outline the structure of the proof using an example. The formal argument is presented in the Appendix.

Suppose $n = 13$. Let $q = P(I) = 1/2$; then the statistical rule says to convict if and only if more than half the jurors have a guilty signal, so $R^S(13) = 7$. Suppose that $R = 9$, hence R is suboptimal. Let Juror 13 deviate from sincere voting and vote Convict regardless of his signal. Is this enough to restore the incentives of other jurors to vote sincerely? No, because now to convict we need eight out of the first 12 jurors to vote Convict, while $R^S(12) = 7$. So, let also Juror 12 deviate to always voting Convict. Now, to convict we need seven out of the first 11 jurors to vote Convict, which is still not the statistical rule. So, it is not an equilibrium for the remaining 11 jurors to vote sincerely. This procedure continues until we reach the point where Jurors 10–13 always vote Convict, and Jurors 1–9 vote sincerely. To convict we need at least five out of the first nine “sincere voters” to vote Convict, which is the statistical rule for nine jurors. Hence, it is a best response for Jurors 1–9 to vote sincerely at this equilibrium. What about Jurors 10–13? If one of them deviates to voting sincerely, then he is voting sincerely under a rule which is not the statistical one for the number of players, so he is better off always voting Convict. ||

9. There are many other Nash equilibria, for instance, those where no juror is ever pivotal.

One implication of this proposition is that, at least in the class of monotone pure strategies equilibria, we can quantify the inefficiency induced by a suboptimal voting rule. The efficiency of a jury with a suboptimal voting rule is equal to the efficiency of a smaller jury endowed with the optimal voting rule.

We remark that $R^S(n)$ spans all numbers between zero and $n + 1$ as q and $P(I)$ vary in $[0, 1]^2$. Thus, with a given number exogenously informed jurors, any voting rule (unanimity, simple majority, and anything in between) can be optimal, depending on the jurors' cost of mistakes and prior probabilities. This will not be the case when the number of informed jurors is endogenous. Suppose, on the other hand, we fix the jurors' cost of mistakes and prior probabilities, and provide the mechanism designer with a supply of N exogenously informed jurors. Then the optimal jury size is N , with larger values of N giving rise to more informed decisions. Furthermore, if we consider exogenous increases in the number N of informed jurors, we obtain a limit result concerning the optimal voting rule. The voting rule converges to simple majority as the number of informed jurors grows large, irrespective of q and $P(I)$ (see Lemma 1). The ratio R/n thus converges to $1/2$, and is within the interval $[1 - p, p]$ for N larger than some number. This property will be important to understand the intuition behind Theorem 1.

4. EQUILIBRIUM WITH INFORMATION ACQUISITION

In this section we look for the optimal voting rule with information acquisition. In designing the optimal rule, we allow the designer to choose the number of jurors n . The mechanism designer wants to have the largest number of jurors subject to the constraint that they be willing to become informed.

As usual in voting games, given any voting rule there are many equilibria. We focus on equilibria where agents play pure strategies. This allows us to restrict attention to equilibria where all jurors get informed. Indeed, if there were jurors who do not get informed at some equilibrium, the mechanism designer could offer another mechanism to only those jurors who get informed, replicating the original mechanism and saving on the cost λ .

Once we restrict attention to rules where all n jurors get informed, it is easy to characterize the optimal rule for given n . For all jurors to be willing to acquire information in the first stage, they all must be voting sincerely in the second stage. By Proposition 1, the only rule under which jurors vote sincerely is the statistical rule $R^S(n)$. Hence, given any number n , the only mechanism that stands a chance of inducing information acquisition on the part of the n jurors is the statistical rule $R^S(n)$.¹⁰ This is established in Part 1 of the following theorem.

Provided that the term $(n^* + 1)/n^*$ is close to 1, we can approximately state Part 2 as: "the optimal fraction of votes required to convict belongs to the interval $(1 - p, p)$ ". In other words, the optimal voting rule can never be closer than approximately $1 - p$ to unanimity or veto power. Using the definition of status quo, we can rephrase this statement. Suppose the status quo is Acquittal, *i.e.* the optimal voting rule requires at least a simple majority of "convict" votes to convict;¹¹ then Part 2 says that the optimal voting rule cannot require more than approximately pn "convict" votes to convict. If the status quo is Conviction, then the optimal voting rule can't require more than approximately pn "acquittal" votes to acquit.

Theorem 1. *Assume agents play an equilibrium in monotone pure strategies. Given $c > 0$, $P(I)$, p , q , denote with n^* the optimal number of jurors and with R^* the optimal voting rule. Suppose $n^* > 0$. Then*

10. Of course, given n it may be that c is too high and therefore it is not an equilibrium for all jurors to acquire information even under the statistical rule $R^S(n)$.

11. From the definition of status quo it follows that $R^S(n) > n/2$.

- (1) $R^* = R^S(n^*)$.
 (2) n^* is such that $(1 - p)p < \frac{n^*+1}{n^*} \frac{R^S(n^*)}{n^*+1} \left[1 - \frac{R^S(n^*)}{n^*+1} \right]$.

Proof. Part 1: At the equilibrium of the optimal mechanism, all jurors acquire information. Indeed, take an equilibrium of a mechanism where some agents do not get informed with some probability. Because we are at a pure strategy equilibrium, these agents do not get informed with probability one. Then the equilibrium of the voting game is as described in Proposition 2. Thus, one can construct another mechanism which is equivalent in terms of outcomes, where the role of the uninformed agents is played by the mechanism itself, and all agents who get informed play the same strategy as before. This new mechanism gives the same expected utility to the mechanism designer, and saves on the lexicographic cost λ . Thus, the original mechanism is dominated.

If all jurors acquire information, it must be that they expect to make use of this information in the voting stage. Thus, jurors must be voting sincerely at equilibrium. Then Proposition 1 yields the result.

Part 2: Since c is positive, the optimal value n^* is chosen so that the marginal incentive for jurors to stay informed is greater than c at n^* , and smaller at $n^* + 2$. Consider a putative equilibrium where all n jurors are informed and $R = R^S(n)$. By Proposition 1, an informed juror votes sincerely and nets a payoff (gross of information cost) of $\Pi(R, n)$. An uninformed juror can vote Convict, in which case the payoff is $\Pi(R - 1, n - 1)$; or Acquit, in which case his payoff is $\Pi(R, n - 1)$. Therefore, a juror's marginal incentive to stay informed is

$$MISI(R, n) = \min\{\Pi(R, n) - \Pi(R - 1, n - 1), \Pi(R, n) - \Pi(R, n - 1)\}.$$

At the optimal n , $MISI(R^S(n + 2), n + 2) - MISI(R^S(n), n)$ must be negative, for otherwise it would be possible to increase n to $n + 2$ while maintaining the jurors' incentives to acquire information. From Lemma A4 in Appendix A, $MISI(R^S(n + 2), n + 2) - MISI(R^S(n), n)$ is negative if and only if

$$(1 - p)p < \frac{R^S(n)[n + 1 - R^S(n)]}{(n + 1)n}.$$

Rearranging the L.H.S. yields the result. \parallel

The conclusion of Part 1 continues to hold in a model where information can be acquired continuously. Suppose p was chosen continuously at an increasing cost $c(p)$. We now show that, if in a monotonic equilibrium a juror acquires *any* amount of information $p > 0$, then that juror must be voting sincerely. Suppose a juror is not voting sincerely: then either he is playing a strategy that is independent of his signal, or he is randomizing in at least one state.¹² In both cases, another best response in the voting game is to vote independent of his signal, a strategy that can be implemented more cheaply because it does not necessitate acquiring any information. Thus, if the juror is acquiring any amount of information $p > 0$, he must be planning to vote sincerely. Then Proposition 1 says that the only voting rule that guarantees sincere voting by all jurors is the statistical rule. Thus, the logic of part 1 of Theorem 1 does not rely on the lumpiness of information. In the continuous information model, if the rule is not the statistical rule, then at least one juror will acquire *zero* information.

If jurors were able to exchange information at the voting stage, Theorem 1 Part 1 would be trivial, since jurors would always implement the optimal (statistical) voting rule. Instead, the result is interesting because we assume that jurors cannot communicate at the voting stage.

12. "Inverted" strategies where the juror votes the opposite of his signal are not admissible in monotonic equilibria.

Under this assumption, one may suspect that the mechanism designer might ameliorate the incentives to acquire information by committing to a voting rule that is suboptimal given the equilibrium number of informed jurors. However, Part 1 of the previous theorem guarantees that this is not the case, because choosing a suboptimal rule will—at least, under our restriction to equilibria in monotone pure strategies—cause some jurors to forego information acquisition.

Part 2 of Theorem 1 is a necessary condition for optimality, that links the voting rule to the quality of information that is available to jurors. The first observation is a cautionary point about the way we interpret the result. When the term $(n^* + 1)/n^*$ is close to 1 then it is appropriate to state Part 2 as: “the optimal fraction of votes required to convict belongs to the interval $(1 - p, p)$ ”. However, when that term is large (that is, when n^* is small), our necessary condition is less strong.

The relationship in Part 2 imposes some restrictions on the minimum quality of information that can justify a given voting rule as optimal. As an illustration of this condition, suppose we observe a requirement of unanimity among thirteen jurors for conviction, and we believe this to be an optimal mechanism. Then, Part 2 yields that the quality of information p must be such that

$$(1 - p)p < \frac{13 + 1}{13} \frac{13}{13 + 1} \left[1 - \frac{13}{13 + 1} \right] = 1/14,$$

so $p > 0.92258$.

The proviso in Part 2 that n^* be positive takes care of the fact that, if c is too large, it is not possible to induce any juror to become informed. If, however, c is sufficiently low that some jurors can be induced to become informed, then our result applies. The bound derived in Part 2 places no restriction on the parameters q and $P(I)$. This is interesting because changing q and $P(I)$ affect $R^S(n)$. We know, for instance, that for fixed number n of informed jurors, choosing q large enough guarantees that unanimity is the optimal rule. Nevertheless, we have shown that when information is endogenous, the bounds given in Part 2 hold irrespective of q . Why is this the case? The reason is that when information is endogenous, at an optimum the incentives to stay informed must be decreasing in the number of jurors. When the voting rule is consensual (too close to unanimity, as measured by the inequality in Part 2), the incentives to acquire information increase with the number of jurors; informed jurors, that is, are a complementary good. This cannot happen if committee size is chosen optimally, because it implies that it is possible for the mechanism designer to change (increase) committee size without depressing the jurors’ incentives to acquire information and thus achieve a more informed decision. Therefore, at the optimal committee size the condition in Part 2 must hold. Of course, if q is chosen sufficiently large then the optimal committee size will be zero and the optimal rule will be to always acquit.¹³

To understand how the incentives to acquire information change with the number of jurors, observe that they are proportional to the probability of being pivotal. Given a voting rule R , jurors are pivotal only when there are exactly $R - 1$ guilty signals out of n realizations. The probability of exactly $R - 1$ guilty signals decreases as n increases, because any particular realization of signals becomes unlikely as n grows. This suggests that the probability of being pivotal must decrease (**effect 1**). However, notice that R itself moves with n (**effect 2**); if the optimal rule with n jurors is R , when there are $n + 2$ jurors the optimal rule becomes $R + 1$ (see Lemma 1). Now, assume that unanimity rule is optimal with n jurors. As we increase the size of the jury to $n + 2$ and we correspondingly adjust the voting rule, the ratio *Rule/number* moves closer to $1/2$, and hence to p , if p is close to $1/2$. This increases the chance that jurors

13. If q is closer to 1 than some threshold \bar{q} , then the rule “always acquit” is first-best optimal. Regardless, that is, of the positive externality of information acquisition, the cost of even one juror becoming informed may exceed the expected social benefit of information. The threshold \bar{q} is an increasing function of the number of agents who benefit from an accurate decision.

are pivotal, because percentages of guilty signals close to p are more likely (happen more often) than percentages close to 1. Thus, effect 2 works towards increasing the probability of being pivotal. Part 2 of Theorem 1 shows that effect 2 dominates effect 1. Thus, when an increase in the number of jurors pushes R/n down towards p (or up towards $1 - p$), the incentives of all jurors to acquire information increase.

It is important to realize that the incentives to stay informed are not monotonic in n . Indeed, as the jury size increases and R/n converges to $1/2$, there comes a point where R/n falls below p . From that point on, both effects 1 and 2 work in the same direction, and increasing jury size decreases the incentives to stay informed. When the incentives to stay informed fall below the cost c , further increasing the jury size causes some jurors to forego information acquisition.

Our assumption is that the mechanism designer ignores the cost of jurors acquiring information, or, equivalently, does not compensate jurors for their cost. An alternative assumption is that the mechanism designer maximizes the social value of the decision minus the social cost of information. The social value is at least the sum of the utilities of the jurors who sit in the jury, and possibly of others *potential* jurors—those who are not jurors, but might be. So, in a jury size of n , the mechanism designer has cost of type I and II errors of vq and $v(1 - q)$, respectively, where $v \geq n + \{\text{\# of potential jurors}\}$. The mechanism designer also takes into account the cost of information that every juror incurs in equilibrium (if jurors are actually to be compensated for the information cost, then the way to do this is to offer a compensation for participation, since obviously we cannot condition the compensation on the unobservable information acquisition). This alternative specification does not affect the main force driving part 2 of Theorem 1, namely, that if the rule R is consensual and if the mechanism designer finds it beneficial to have a jury with n members, then the mechanism designer will want to increase the jury size to $n + 2$. Indeed, in part 2 we have proved that increasing the jury size to $n + 2$ increases the jurors' incentives to acquire information. Now, if a juror finds it worthwhile to acquire information then *a fortiori* the mechanism designer would want him to, even if this means compensating the juror for the cost of information. This is because, when deciding whether to acquire information, the juror trades off a cost of size c with a benefit of size q . In contrast, the mechanism designer trades off the cost c with a benefit of size $vq > q$. This shows that the basic conclusion of the paper is robust to this alternative specification.

5. COMPARATIVE STATICS

In this section we provide some comparative statics results on optimal jury size. All results refer to the most efficient equilibrium in monotone pure strategies that was investigated in the previous section.

The first result is that the optimal jury size is nonincreasing in c , the cost of information. This is because if a given jury size n is sustainable with cost c , then it is also sustainable with cost $c' < c$. This tells us that committee size—and hence the efficiency of the decision—is nonincreasing in c .

Fact: The optimal committee size and the efficiency of the decision is nonincreasing in c .

Less obvious is how the optimal size of the jury changes as q , $P(I)$ and p vary. Although the optimal jury size is not monotonic due to integer problems, we can nonetheless provide some comparative statics results on the optimal jury size.

Definition 3. Consider two numbers x and y in $(0, 1)$. We say that x is p -larger than y if $\frac{x}{1-x} \left(\frac{1-p}{p}\right)^2 > \frac{y}{1-y}$. We say that x is p -far from y if either x is p -larger than y or y is p -larger than x .

If x is p -far from y then either x is sufficiently smaller than y so that $\frac{1-y}{y} < \frac{1-x}{x} \left(\frac{1-p}{p}\right)^2$, or x sufficiently larger than y so that $\frac{x}{1-x} \left(\frac{1-p}{p}\right)^2 > \frac{y}{1-y}$. The significance of this definition is related to the statistical problem. Suppose q is p -far from q' . Then given a decision problem with n signals, the statistical rule when cost of mistake is q is different than the statistical rule with cost of mistake q' .

The next proposition shows that, when the size of the jury is large, making the decision problem more balanced, that is, either (a) making the costs of mistake more similar, or (b) making the prior more diffuse, increases the incentives to acquire information and therefore optimal jury size.

Proposition 3. *Consider any \hat{q} which is p -far from $1/2$, and fix any c sufficiently small. Then there is a set \mathcal{Q} of costs of mistake which are closer to $1/2$ than \hat{q} , such that the optimal committee size is larger for any $q' \in \mathcal{Q}$ than for \hat{q} .*

Similarly, consider any $P(I)$ which is p -far from $1/2$, and choose any c sufficiently small. Then there is a set \mathcal{P} of prior beliefs which are closer to $1/2$ than $P(I)$ such that the optimal committee size is larger for any prior in \mathcal{P} rather than for $P(I)$.

Proof. See Appendix A. ||

Some intuition for this result may be gained by noticing that the incentives to become informed must equal zero when q equals zero or one. In this case the optimal committee size is zero. If we start from the two extremes $q = 0$ or 1 , therefore, the incentive to become informed and optimal committee size must be increasing as q moves towards $1/2$, consistent with the statement in Proposition 3.

We can also obtain a comparative statics result on optimal committee size as the informativeness of the private signal declines. The flavour is that when information is less accurate, committees are larger. This finding parallels some findings of Moscarini and Smith (2000), who study the value of information and rank information systems in decision problems in which the cost of information is small.

Proposition 4. *Consider any $p < \hat{p} < 1/2$. Given any c sufficiently small, the optimal committee size is larger or equal at p than at \hat{p} . Furthermore, for small c there is a set \mathfrak{P} of accuracies which are smaller than \hat{p} , such that the optimal committee size for any $p' \in \mathfrak{P}$ is strictly larger than for \hat{p} .*

Proof. See Appendix A. ||

Figure 1 shows the marginal incentives to stay informed as a function of p in a committee with $q = 0.9$ and $P(I) = 1/2$.

The thin solid curve represents the incentive to stay informed in a committee of 13 members. Each peak in the curve corresponds to a different rule R , each optimal given p ; its tallest peak, for example, corresponds to $R = 9$; the one to its right to $R = 8$. For given p , a committee size of 13 is feasible (that is, all committee members are willing to acquire information) only if the cost of information is below this curve. To illustrate what happens when the cost of information decreases, we have depicted the marginal incentive to stay informed in a committee of size 15 with the thin dashed curve in Figure 1. This curve is higher than the solid thin curve when p is small (here, when $p < 0.61$). Thus, when $p < 0.61$ a jury of size 13 could not be optimal since if it is feasible, then a larger jury of 15 is also feasible and dominates a jury of

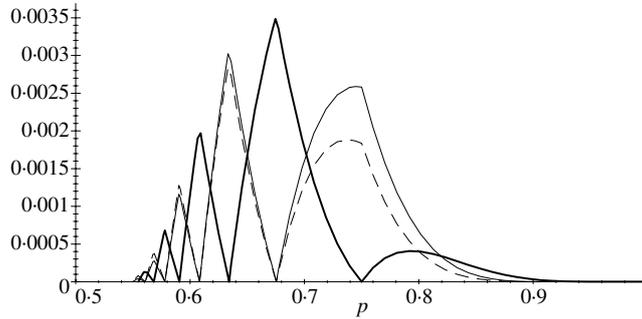


FIGURE 1
Marginal incentive to stay informed as a function of signal accuracy

size 13 (see Theorem 1). If, conversely, a jury of size 13 is optimal, p must be larger than 0.61. Suppose that this is the case. Then, as the cost of information decreases, at some point a jury of size 15 becomes feasible.

To illustrate Proposition 4, we depict with the thick solid curve the incentive to stay informed in a 14-member committee. We first deal with the statement involving p' (the second one in the proposition). Fix any \hat{p} greater than approximately 0.68: the thin line evaluated at \hat{p} lies below the thick line evaluated at $p' = 0.68$. Thus, whenever the accuracy of information exceeds 0.68, if a committee of size 13 is feasible, then a larger committee is feasible for a lower accuracy. The range $\hat{p} < 0.61$ is not relevant since, as was argued above, a committee of size 13 cannot be optimal for that accuracy. Finally, we need to deal with the range of accuracies between 0.61 and 0.68. In that range, decreasing the accuracy of the signal will result in a lower committee size. As the cost of information becomes smaller, however, and committee sizes larger than 13 become feasible, the interval (0.61, 0.68) behaves like the ones to its right, and the highest value of the thin curve in that interval becomes smaller than the highest value of the thick curve in the interval (0.59, 0.63). The first statement in Proposition 4, the one involving p , is of the form “for every p and \hat{p} there exists a $c \dots$ ”. In Figure 1, c is implicitly fixed by the fact that we are looking at jury sizes around 13. Thus, the statement is validated only for some values of p and \hat{p} , but not for all. As c becomes smaller, larger committee sizes become feasible and the set of values of p and \hat{p} for which the statement holds true will increase, eventually to include the entire interval $[1/2, 1]$.

The fact that the incentives to acquire information may increase as the quality of information goes down may seem puzzling, because in a decision problem the value of coarser information cannot exceed that of more accurate information. However, in our game we must consider the fact that when information becomes less accurate *all* jurors receive a coarser signal. This is equivalent to a more noisy behaviour on the part of all jurors. This effect may actually increase the probability that jurors are pivotal, and thus the incentives to acquire information. Some intuition may be obtained by noticing that as p decreases, R becomes more extreme (*i.e.* closer to unanimity if it is greater than $n/2$). Thus, ignoring the change in n , decreasing p makes the ratio R/n closer to p . This effect tends to increase the probability that a juror is pivotal, strengthening the incentives to become informed. The effect of noise on the incentives to acquire information is discussed further in the Conclusions.

6. ASYMPTOTICALLY EFFICIENT RULES

Lemma 1 has some similarity with Condorcet’s Jury theorem. Under the assumption of sincere voting, Condorcet’s theorem states that “... the probability that the group, deciding on the basis

of majority rule, makes the correct decision [...] increases as the size of the group increases and quite rapidly approaches perfection” (Miller, 1986, pp. 174,175). Thus, Condorcet’s theorem asserts that majority rule is asymptotically optimal. However, asymptotic optimality is a relatively weak restriction, in the sense that many other rules besides simple majority are asymptotically optimal; for instance, all the rules where, as n grows, R/n approaches a number between $1 - p$ and p , are also asymptotically optimal. In contrast, our result characterizes the asymptotic behaviour of the optimal (for given n , not asymptotically) decision rule.

A stronger result concerning asymptotically optimal rules is that of Feddersen and Pesendorfer (1997). The example outlined in the proof of Proposition 2 affords a simple intuition for their result. Feddersen and Pesendorfer study symmetric (mixed strategy) equilibria in large juries where jurors are exogenously informed. They show that any rule R that can be expressed as $R = k \cdot n$ for some $k \in (0, 1)$ aggregates information perfectly as n goes to infinity. Take our example, and let n and R grow so as to keep $R = (9/13)n$. Reasoning as in the above example, there is an equilibrium where a fraction s of the jurors vote sincerely (Jurors 1–9 in the example), and the remaining $(1 - s)$ of the jurors vote to rectify the suboptimal rule (Jurors 10–13 in the example, who vote Convict regardless of their signal). The outcome is equivalent to that of a smaller jury of size sn which decides by simple majority rule. As n converges to infinity, such a jury aggregates information perfectly.

In the context of the same model, Feddersen and Pesendorfer (1998) show that unanimity can “never be optimal”. More formally, Feddersen and Pesendorfer show that unanimity is a suboptimal rule as we fix $P(I)$ and q , and let n go to infinity. It is interesting to contrast their result to ours. To do this, first we present a pure strategy version of their result in an example. Consider the most efficient pure strategy equilibrium of a voting game with $n > 2$, $q = P(I) = 1/2$, and voting rule equal to unanimity. In view of Proposition 1, simple majority is the only rule under which jurors are willing to vote truthfully. Then, given unanimity rule, only Jurors 1 and 2 can vote informatively at equilibrium; they decide the outcome according to “simple majority between the two of them”. Jurors 3 through n always vote Convict.¹⁴ The efficiency of this jury with unanimity is equal to the efficiency of a two-member jury with simple majority rule. An increase in n only has the effect of increasing the number of those jurors who vote uninformatively, and thus has no informational benefit.

The result in Feddersen and Pesendorfer (1998) is therefore about the *asymptotic* inefficiency of *some* suboptimal mechanisms (unanimous rules). Given $P(I)$ and q , there are many more mechanisms which are suboptimal for given n , but are efficient asymptotically (when n goes to infinity). Among those are almost-unanimous rules such as $R = 0.99 \cdot n$. Feddersen and Pesendorfer accept these rules as “good”, because asymptotically efficient.¹⁵ In contrast, we are able to rule out such rules (if $p < 0.99$). This is because our result characterizes optimal (not only asymptotically optimal!) mechanisms. Furthermore, notice that it is crucial for their result that the number of jurors converges to infinity while all other primitives are fixed. If we reverse the order of taking limits, we find that for any finite size of jury it is possible to find sufficiently extreme preferences such that unanimity is the optimal rule. As we have seen, this is not possible in our model unless the quality of information is very high. This is not a criticism of Feddersen and Pesendorfer, but rather a remark on the model with exogenously informed jurors, where jury size is an exogenous parameter.

14. This is an application of Proposition 2. To see that it is a best response for jurors 3... n to always vote Convict, consider the change in outcome that would result from juror 3 voting sincerely. The only case where the outcome changes is when he is pivotal. If juror 3 is pivotal it means that jurors 1 and 2 are voting Convict, *i.e.* both have received a g signal. In this case, juror 3 prefers to vote Convict regardless of his signal.

15. As a side remark, this way of evaluating mechanism is quite “discontinuous”, in some intuitive sense, in the space of mechanism.

7. MONETARY TRANSFERS

Until now, we have examined a problem where the mechanism designer’s only instruments are the size of the jury and the voting rule. Implicitly, we have forbidden monetary transfers.¹⁶ This is appropriate to model many situations where monetary incentives are not allowed. We now show that there is a theoretical reason to restrict to that case: the full information outcome can trivially be attained if monetary transfers are allowed.

Consider any number $n > 0$ of jurors; jurors may acquire signals $x_i \in \{i, g\}$, and they report $\hat{x}_1 \dots \hat{x}_n$. Let $\hat{x}_i \in \{0, 1\}$, with the convention that 0 means innocent and 1 means guilty. The mechanism designer wants to implement some decision rule $d(x_1, \dots, x_n) \in \{\text{Convict}, \text{Acquit}\}$. The mechanism designer can set up a transfer scheme $T_1(\hat{x}_1, \dots, \hat{x}_n), \dots, T_n(\hat{x}_1, \dots, \hat{x}_n)$.

Timing:

- (1) The mechanism designer picks n jurors, and sets the decision rule d and the transfer scheme T_1, \dots, T_n .
- (2) Jurors observe n, d and $T_1 \dots T_n$. Then, jurors choose whether to acquire information at cost c .
- (3) Jurors report $(\hat{x}_1, \dots, \hat{x}_n)$.
- (4) The mechanism designer operates transfers $T_1(\hat{x}_1, \dots, \hat{x}_n), \dots, T_n(\hat{x}_1, \dots, \hat{x}_n)$ and implements $d(\hat{x}_1, \dots, \hat{x}_n)$.

By choosing the transfer scheme T appropriately, the mechanism designer can induce the jurors to report their true signals, regardless of the specification of d .

To simplify the analysis, assume $P(I) = 1/2$. Let n be even. Set

$$T_i(\hat{x}_1, \dots, \hat{x}_n) = \begin{cases} K & \text{if } \text{sgn}\{\hat{x}_i - \frac{1}{2}\} = \text{sgn}\left\{\left(\sum_{j \neq i} \hat{x}_j\right) - \frac{n-1}{2}\right\} \\ -K & \text{otherwise.} \end{cases}$$

Thus, an agent gets a transfer of K if more than half the reports of the other agents agree with his report, and $-K$ otherwise. Under the assumption that the other jurors report truthfully, a juror with signal g deems it more likely that the true state is G and thus more likely that the majority of jurors reports 1. This gives him and incentive to report 1, *i.e.* report truthfully, to maximize the chance of receiving K . Setting K large ensures that the percentage of a juror’s payoff that depends on d is small; that is, the juror’s decision is guided solely by the concern of earning a large transfer. This shows that, for K large enough, reporting truthfully is an equilibrium. This procedure allows to extract the signals from the jurors, regardless of the policy d that is implemented.

For K large enough, all jurors will want to acquire information in stage 2, since the voting stage is very risky. Thus, this procedure induces all jurors to acquire information and then report it truthfully. Since this can be done for any n , the mechanism designer can reach an arbitrarily good decision by calling up large numbers of jurors.

Finally, we can modify this game so that the expected transfer is zero. This is done by imposing a tax on the jurors in stage 1, of an amount equal to the expected revenue that they expect to make from the transfer.

16. Except perhaps as lump sums refunds for participating to the committee meeting. Such transfers are just a rescaling of the juror’s utility and have no implications for incentives.

8. CONCLUSION

We have analysed the design of institutions for collective decisions, such as management teams, committees, and juries. We view these as situations where the information acquisition activity is a fundamental part of the decision process. We have posed questions such as: under what circumstances should collective decision making rely on majority decisions, and when should it require more stringent measures of consensus, such as supermajorities or unanimity? How large should teams or juries be?¹⁷

We have explored the answers to these questions within a model of private production of information. In our model, agents privately gather costly information, and then aggregate it to produce a collective decision. Because information is a public good, it will be underprovided relative to the social optimum. Our problem is therefore one of team effort, where the mechanism designer can modify the production function to achieve higher effort.

We have characterized the voting mechanism that produces the most informed decision. We have obtained a necessary condition for the voting rule to be “consensual”, *i.e.* close to unanimity or veto power: the optimal voting rule cannot be close to being consensual unless the accuracy of the information available to individual committee members is high. Our analysis suggests that consensus-based decision making is suboptimal in environments where individual information is coarse, precisely those environments where collective decision making is thought to be most beneficial. We have also provided comparative statics results on the optimal size of the committee.

Our results are conceptually different than what can be obtained in a model with exogenously informed jurors. In such a model, Feddersen and Pesendorfer (1997, 1998) show that as the jury size increases, unanimity is inefficient, but rules requiring 99% of the votes to convict are efficient (in the limit). In contrast, we are able to exclude such rules as suboptimal, so long as the jurors’ information is less than 99% accurate.

An important qualification of our results is that we have restricted attention to equilibria in pure strategies. In particular, we have ignored equilibria in which some jurors do not acquire information and then mix between convicting and acquitting. While mixing reduces the efficiency of the aggregation of information, it can increase the incentives to acquire information of those players who do not mix.¹⁸ There are circumstances in which a bit of this mixing may actually improve on the pure strategy equilibrium, in that the gain in incentives exceeds the loss in information aggregation properties. This suggests looking for a more general form of mechanism, one where the rule is a random variable \mathcal{R} (and in equilibrium all jurors get informed). One expects that the force identified in this paper, namely, the gain to increasing the jury size when \mathcal{R} is extreme, remains valid in this context. The determination of the optimal \mathcal{R} is an interesting question which is left for future research.

An interesting question is the role of communication among committee members. In order to focus on the incentives to acquire information, in this paper we have ignored the scope for communication. While communication may be costly or imperfect in a number of practical situations, partial communication can occur among committee members even when they have

17. See Haleblan and Finkelstein (1993) for a review of the theoretical and empirical literature on team size. Concerning jury size, the U.S. Supreme Court holds that juries of more or less than 12 jurors are constitutional, but there is a minimum of six jurors for nonpetty criminal cases. Today, over thirty U.S. states allow juries of less than 12 members. See Abramson (1995, p. 180, 181). For a review of the debate on unanimity in criminal juries, see Abramson (1995, Chapter 5).

18. For example, consider the constellation of parameters $q = P(I) = 1/2$ and $p = 0.9$. In this setup, consider: jury A, where $R = 4$ and there are seven members, five of which vote sincerely and the rest randomize with equal probability between convict and acquit; and jury B, where $R = 3$ and there are five members all of whom vote sincerely. Then, it can be shown that an informed juror’s incentives to stay informed are larger in jury A. I am grateful to an anonymous referee for pointing out the importance of mixed strategies and for providing this example.

heterogeneous preferences. This is the finding of Doraszelsky *et al.* (1999), who investigate the role of communication in a 2-members committee. The fact that jurors may communicate ought to place some constraint on the mechanism designer's effectiveness in designing the mechanism. Gerardi and Yariv (2002) make progress in studying the effect of communication on the incentives to acquire information.

APPENDIX A. PROOFS

Proof of Proposition 1

Proof. Let us construct the most efficient pure strategy equilibrium. Suppose $R^S(n) > 1 + n/2$ (the case where $R^S(n) < 1 + n/2$ is treated in exactly the same way). Remember, from Lemma 1, that $R^S(n)$ is increasing at a speed of $n/2$.

Case A. $R < R^S(n)$.

Start with $n_1 = 0$, and progressively increase n_1 until one of the two conditions below is satisfied.

Condition 1: $R^S(n - n_1) = n - n_1 + 1$. In this case, the best pure strategy equilibrium is for everyone to vote Acquit regardless of their signal.

Condition 2: $R^S(n - n_1) = R$. In this case, there is an equilibrium where jurors $1 \dots n_1$ vote Acquit regardless of their signal, and jurors $n_1 + 1 \dots n$ vote sincerely. The outcome at this strategy profile is equivalent to the outcome of a game with $n - n_1$ jurors who vote sincerely. Thus, by Proposition 1 it is a best response for jurors $n_1 + 1 \dots n$ to vote sincerely. Next, consider a deviation by a juror among $1 \dots n_1$. Suppose he deviates to voting sincerely; then this is equivalent to a voting game with $n - n_1 + 1$ jurors and rule R which, by construction of n_1 , is smaller than $R^S(n - n_1 + 1)$. Since

$$\Pi(R, n) - \Pi(R, n - 1) = -\frac{R}{n}[\Pi(R + 1, n) - \Pi(R, n)], \quad (\text{A.1})$$

whenever R is smaller than the optimal rule $R^S(n - n_1 + 1)$, we have $\Pi(R, n - n_1 + 1) < \Pi(R, n - n_1)$, *i.e.* the payoff from voting sincerely is smaller than the payoff from always voting Acquit. This shows that it is not profitable for jurors $1 \dots n_1$ to deviate to voting sincerely. *A fortiori*, it is not profitable to deviate to always voting Convict. Thus, it is a best response to always vote Acquit.

Case B. $R > R^S(n)$.

This case is treated symmetrically to Case A, except that here jurors who ignore their information will vote Convict. Notice that in Case B the case treated in Condition 1 cannot happen.

Finally, this is the most efficient pure strategy equilibrium because it is the equilibrium where the greatest number of jurors vote sincerely. \parallel

Proof of Lemma 5

We break the proof into a series of lemmas.

The marginal incentive to stay informed is

$$MISI(R, n) = \min\{\Pi(R, n) - \Pi(R - 1, n - 1), \Pi(R, n) - \Pi(R, n - 1)\}.$$

Lemma A1. $MISI(R, n) = \min\left\{-\frac{R}{n}[\Pi(R + 1, n) - \Pi(R, n)], \frac{n-R+1}{n}[\Pi(R, n) - \Pi(R - 1, n)]\right\}$.

Proof. Let us start by observing that

$$\begin{aligned} & \Pi(R + 1, n) - \Pi(R, n) \\ &= qP(I) \binom{n}{R} (1-p)^R p^{n-R} - (1-q)P(G) \binom{n}{R} (1-p)^{n-R} p^R \\ &= \binom{n}{R} (1-p)^{n-R} p^R \left[qP(I) \left(\frac{1-p}{p}\right)^{2R-n} - (1-q)P(G) \right], \end{aligned}$$

whence

$$\Pi(R, n) - \Pi(R - 1, n) = \binom{n}{R-1} (1-p)^{n-R+1} p^{R-1} \left[qP(I) \left(\frac{1-p}{p}\right)^{2R-n-2} - (1-q)P(G) \right] \quad (\text{A.2})$$

and

$$\begin{aligned} & \Pi(R, n-1) - \Pi(R-1, n-1) \\ &= \binom{n-1}{R-1} (1-p)^{n-R} p^{R-1} \left[qP(I) \left(\frac{1-p}{p} \right)^{2R-n-1} - (1-q)P(G) \right]. \end{aligned} \quad (\text{A.3})$$

(1) Derivation of the term $\Pi(R, n) - \Pi(R, n-1)$.

The term $\Pi(R, n+1) - \Pi(R, n)$ represents the change in expected utility from adding an additional juror, as we keep fixed the rule R . We wish to show that

$$\begin{aligned} \Pi(R, n+1) - \Pi(R, n) &= P(I) \binom{n}{R-1} (1-p)^{R-1} p^{n-R+1} (-q)(1-p) \\ &+ P(G) \binom{n}{R-1} (1-p)^{n-R+1} p^{R-1} (1-q)(p). \end{aligned} \quad (\text{A.4})$$

To this end, write

$$\begin{aligned} & \Pi(R, n+1) - \Pi(R, n) \\ &= -qP(I) \left[\sum_{x=R}^{n+1} \binom{n+1}{x} (1-p)^x p^{n+1-x} - \sum_{x=R}^n \binom{n}{x} (1-p)^x p^{n-x} \right] \\ & - (1-q)P(G) \left[\sum_{x=0}^{R-1} \binom{n+1}{x} (1-p)^{n+1-x} p^x - \sum_{x=0}^{R-1} \binom{n}{x} (1-p)^{n-x} p^x \right]. \end{aligned} \quad (\text{A.5})$$

First, we operate on the term in brackets multiplying $qP(I)$. We wish to verify that for a generic R we have

$$\sum_{x=R}^{n+1} \binom{n+1}{x} (1-p)^x p^{n+1-x} - \sum_{x=R}^n \binom{n}{x} (1-p)^x p^{n-x} = \binom{n}{R-1} (1-p)^R p^{n-R+1}. \quad (\text{A.6})$$

We do this by induction. When $R = n+1$ equality (A.6) holds, as both sides equal $(1-p)^{n+1}$. Let us assume then that equality (A.6) holds for $R+1$, and let us check that it holds for R . The L.H.S. of (A.6) can be factorized as

$$\begin{aligned} & \binom{n+1}{R} (1-p)^R p^{n+1-R} - \binom{n}{R} (1-p)^R p^{n-R} \\ & + \left[\sum_{x=R+1}^{n+1} \binom{n+1}{x} (1-p)^x p^{n+1-x} - \sum_{x=R+1}^n \binom{n}{x} (1-p)^x p^{n-x} \right] \\ & = \binom{n+1}{R} (1-p)^R p^{n+1-R} - \binom{n}{R} (1-p)^R p^{n-R} + \left[\binom{n}{R} (1-p)^{R+1} p^{n-R} \right], \end{aligned}$$

where the equality follows from the inductive assumption. Therefore, we can rewrite equation (A.6) as

$$\begin{aligned} & \binom{n+1}{R} (1-p)^R p^{n+1-R} - \binom{n}{R} (1-p)^R p^{n-R} + \binom{n}{R} (1-p)^{R+1} p^{n-R} \\ & = \binom{n}{R-1} (1-p)^R p^{n-R+1}. \end{aligned}$$

Dividing through by $(1-p)^R p^{n-R}$ we get

$$\binom{n+1}{R} p - \binom{n}{R} + \binom{n}{R} (1-p) = \binom{n}{R-1} p. \quad (\text{A.7})$$

Observing that

$$\binom{n}{R-1} = \frac{R}{n-R+1} \binom{n}{R} \quad \text{and} \quad \binom{n+1}{R} = \frac{n+1}{n+1-R} \binom{n}{R},$$

we divide through by $\binom{n}{R}$ in equation (A.7) to obtain

$$\left(\frac{n+1}{n+1-R} \right) p - 1 + (1-p) = \left(\frac{R}{n-R+1} \right) p.$$

This equality is true, which shows that equation (A.6) is verified for any value of $R \leq n+1$.

Now, let us operate on the term in brackets multiplying $(1-q)P(G)$. We wish to show that for a generic R we have

$$\sum_{x=0}^{R-1} \binom{n+1}{x} (1-p)^{n+1-x} p^x - \sum_{x=0}^{R-1} \binom{n}{x} (1-p)^{n-x} p^x \quad (\text{A.8})$$

$$= -\binom{n}{R-1}(1-p)^{n-R+1}p^R.$$

Equation (A.8) clearly holds for $R = 1$ since then both sides of the equality equal $-p(1-p)^n$. So, assume that equality (A.8) holds for $R - 1$, and let us check that it holds for R . Again using the inductive assumption, the L.H.S. of (A.8) can be factorized as

$$\binom{n+1}{R-1}(1-p)^{n-R+2}p^{R-1} - \binom{n}{R-1}(1-p)^{n-R+1}p^{R-1} - \binom{n}{R-2}(1-p)^{n-R+2}p^{R-1}.$$

Substituting into equation (A.8) and dividing through by $\binom{n}{R-1}(1-p)^{n-R+1}p^{R-1}$, we must verify that

$$\left(\frac{n+1}{n-R+2}\right)(1-p) - 1 - \left(\frac{R-1}{n-R+2}\right)(1-p) = -p.$$

This equality is verified, which shows that equation (A.8) is verified for any value of $R \geq 1$.

Substituting expressions (A.6) and (A.8) into expression (A.5) shows that equality (A.4) holds.

From (A.4) we get

$$\begin{aligned} & \Pi(R, n) - \Pi(R, n-1) & (A.9) \\ &= P(I)\binom{n-1}{R-1}(1-p)^{R-1}p^{n-R}(-q)(1-p) \\ & \quad + P(G)\binom{n-1}{R-1}(1-p)^{n-R}p^{R-1}(1-q)(p) \\ &= \binom{n-1}{R-1}(1-p)^{n-R}p^R \left[(1-q)P(G) - qP(I)\left(\frac{1-p}{p}\right)^{2R-n} \right] \\ &= -\frac{R}{n}[\Pi(R+1, n) - \Pi(R, n)]. \end{aligned}$$

(2) Derivation of the term $\Pi(R, n) - \Pi(R-1, n-1)$.

We have

$$\Pi(R, n) - \Pi(R-1, n-1) = [\Pi(R, n-1) - \Pi(R-1, n-1)] + [\Pi(R, n) - \Pi(R, n-1)]$$

Substituting from equation (A.3) and (A.9),

$$\begin{aligned} & \Pi(R, n) - \Pi(R-1, n-1) \\ &= \binom{n-1}{R-1}(1-p)^{n-R}p^{R-1} \left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n-1} - (1-q)P(G) \right] \\ & \quad + \binom{n-1}{R-1}(1-p)^{n-R}p^R \left[(1-q)P(G) - qP(I)\left(\frac{1-p}{p}\right)^{2R-n} \right] \\ &= \binom{n-1}{R-1}(1-p)^{n-R+1}p^{R-1} \left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n-2} - (1-q)P(G) \right] \\ &= \frac{n-R+1}{n}[\Pi(R, n) - \Pi(R-1, n)]. \quad \parallel \end{aligned}$$

Lemma A2. $[\Pi(R+1, n+2) - \Pi(R, n+1)] - [\Pi(R, n) - \Pi(R-1, n-1)] = \frac{n-R+1}{n}[\Pi(R, n) - \Pi(R-1, n)] \left[\frac{(n+1)n}{(n-R+1)R}(1-p)p - 1 \right]$.

Proof.

$$\begin{aligned} & [\Pi(R+1, n+2) - \Pi(R, n+1)] - [\Pi(R, n) - \Pi(R-1, n-1)] \\ &= \frac{n-R+2}{n+2}[\Pi(R+1, n+2) - \Pi(R, n+2)] - \frac{n-R+1}{n}[\Pi(R, n) - \Pi(R-1, n)] \\ &= \frac{n-R+2}{n+2}\binom{n+2}{R}(1-p)^{n-R+2}p^R \left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n-2} - (1-q)P(G) \right] \\ & \quad - \frac{n-R+1}{n}\binom{n}{R-1}(1-p)^{n-R+1}p^{R-1} \left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n-2} - (1-q)P(G) \right] \end{aligned}$$

$$= \frac{n-R+1}{n} [\Pi(R, n) - \Pi(R-1, n)] \left[\frac{(n+1)n}{(n-R+1)R} (1-p)p - 1 \right]. \quad \parallel$$

Lemma A3. $[\Pi(R+1, n+2) - \Pi(R+1, n+1)] - [\Pi(R, n) - \Pi(R, n-1)] = -\frac{R}{n} [\Pi(R+1, n) - \Pi(R, n)] \left[\frac{(n+1)n}{(n-R+1)R} (1-p)p - 1 \right]$.

Proof.

$$\begin{aligned} & [\Pi(R+1, n+2) - \Pi(R+1, n+1)] - [\Pi(R, n) - \Pi(R, n-1)] \\ &= \binom{n+1}{R} (1-p)^{n-R+1} p^{R+1} \left[(1-q)P(G) - qP(I) \left(\frac{1-p}{p} \right)^{2R-n} \right] \\ &\quad - \binom{n-1}{R-1} (1-p)^{n-R} p^R \left[(1-q)P(G) - qP(I) \left(\frac{1-p}{p} \right)^{2R-n} \right] \\ &= -\frac{R}{n} [\Pi(R+1, n) - \Pi(R, n)] \left[\frac{(n+1)n}{(n-R+1)R} (1-p)p - 1 \right]. \quad \parallel \end{aligned}$$

Lemma A4. Suppose $0 < R^S(n) < n+1$. Then $MISI(R^S(n+2), n+2) - MISI(R^S(n), n)$ is negative if and only if

$$(1-p)p < \frac{R^S(n)[n+1-R^S(n)]}{(n+1)n}.$$

Proof. From the previous lemmas, we have that

$$\begin{aligned} & [\Pi(R+1, n+2) - \Pi(R, n+1)] - [\Pi(R, n) - \Pi(R-1, n-1)] \\ &= \frac{n-R+1}{n} [\Pi(R, n) - \Pi(R-1, n)] \left[\frac{(n+1)n}{(n-R+1)R} (1-p)p - 1 \right] \end{aligned}$$

and

$$\begin{aligned} & [\Pi(R+1, n+2) - \Pi(R+1, n+1)] - [\Pi(R, n) - \Pi(R, n-1)] \\ &= -\frac{R}{n} [\Pi(R+1, n) - \Pi(R, n)] \left[\frac{(n+1)n}{(n-R+1)R} (1-p)p - 1 \right]. \end{aligned}$$

Let us evaluate the above quantities at $R = R^S(n)$. Notice that if $0 < R^S(n) < n$ then $R^S(n+2) = R^S(n) + 1$. Thus, if both the above quantities are positive when $R = R^S(n)$, then $MISI(R^S(n+2), n+2) - MISI(R^S(n), n)$ is positive too. In view of the definition of $R^S(n)$ the above quantities have the sign of

$$\frac{(n+1)n}{[n-R^S(n)+1]R^S(n)} (1-p)p - 1.$$

The result then follows immediately. \parallel

Proof of Proposition 3

Proof. We are only going to prove the statement concerning q , since $P(I)$ and q play an interchangeable role in our model. With the obvious meaning of the symbols, we will sometimes attach a subscript q to some quantities. Fix $P(I)$, and let R, n denote the optimal rule and jury size under \hat{q} . We know that $R = R_q^S(n)$, and we denote with \underline{q} and \bar{q} the lowest and highest value of q such that $R = R_q^S(n)$. By definition, $\hat{q} \in [\underline{q}, \bar{q}]$.

We wish to characterize the marginal incentive to stay informed in a jury of size n as q increases within $[\underline{q}, \bar{q}]$. To this end, we notice that $R_q^S(n-1)$ goes from $R-1$ to R as q increases within $[\underline{q}, \bar{q}]$. Therefore, if we denote with \tilde{q} the value at which $R_q^S(n-1)$ switches to R , the marginal incentive to stay informed in a jury of size n is

$$\begin{cases} \Pi_q(R, n) - \Pi_q(R-1, n-1) & \text{for } \underline{q} \leq q < \tilde{q} \\ \Pi_q(R, n) - \Pi_q(R, n-1) & \text{for } \tilde{q} \leq q < \bar{q}. \end{cases}$$

Since

$$\Pi_q(R, n) - \Pi_q(R-1, n-1) = \binom{n-1}{R-1} (1-p)^{n-R+1} p^{R-1} \left[qP(I) \left(\frac{1-p}{p} \right)^{2R-n-2} - (1-q)P(G) \right],$$

for $\underline{q} \leq q < \tilde{q}$ the marginal incentive to stay informed is increasing in q . In fact, it is easy to see that the above quantity assumes value zero at \underline{q} . In contrast, for $\tilde{q} \leq q < \bar{q}$ the marginal incentive to stay informed is

$$\Pi_q(R, n) - \Pi_q(R, n-1) = \binom{n-1}{R-1} (1-p)^{n-R} p^R \left[(1-q)P(G) - qP(I) \left(\frac{1-p}{p} \right)^{2R-n} \right]$$

which is decreasing in q , and assumes value zero at \bar{q} . This shows that the marginal incentive to stay informed in a jury of size n is not monotonic on $[\underline{q}, \bar{q}]$, and that the marginal incentive is single-peaked and maximal at \tilde{q} .

To prove our claim, we need to show that if \tilde{q} is p -smaller than $1/2$ there is a value of q larger than \tilde{q} and a jury size larger than n such that the marginal incentives to stay informed are larger than $\Pi_{\tilde{q}}(R, n) - \Pi_{\tilde{q}}(R-1, n-1)$. In fact, we now show that this is the case for a jury of size $n+1$ with cost of mistake \bar{q} .

By definition of \bar{q} we have $\Pi_{\bar{q}}(R+1, n) - \Pi_{\bar{q}}(R, n) = 0$. This means that $R_{\bar{q}}^S(n+1) = R+1$. Then, the marginal incentive to stay informed in a jury of size $n+1$ with cost of mistake \bar{q} is

$$\begin{aligned} & \Pi_{\bar{q}}(R+1, n+1) - \Pi_{\bar{q}}(R, n) \\ &= \binom{n}{R} (1-p)^{n-R+1} p^R \left[\bar{q}P(I) \left(\frac{1-p}{p} \right)^{2R-n-1} - (1-\bar{q})P(G) \right] \\ &= p \frac{n}{R} \bar{q} \binom{n-1}{R-1} (1-p)^{n-R+1} p^{R-1} \left[P(I) \left(\frac{1-p}{p} \right)^{2R-n-1} - \frac{(1-\bar{q})}{\bar{q}} P(G) \right]. \end{aligned} \quad (\text{A.10})$$

By definition, \bar{q} is such that

$$\frac{P(I)}{P(G)} \left(\frac{1-p}{p} \right)^{2R-n} = \frac{(1-\bar{q})}{\bar{q}},$$

and \tilde{q} is such that

$$\frac{P(I)}{P(G)} \left(\frac{1-p}{p} \right)^{2R-n-1} = \frac{(1-\tilde{q})}{\tilde{q}}.$$

Combining these two equations we get that

$$\frac{(1-\tilde{q})}{\tilde{q}} \frac{1-p}{p} = \frac{(1-\bar{q})}{\bar{q}}. \quad (\text{A.11})$$

Substituting into (A.10) we get

$$\begin{aligned} & \Pi_{\bar{q}}(R+1, n+1) - \Pi_{\bar{q}}(R, n) \\ &= p \frac{n}{R} \binom{n-1}{R-1} (1-p)^{n-R+1} p^{R-1} \bar{q} \left[P(I) \left(\frac{1-p}{p} \right)^{2R-n-1} - \frac{(1-\tilde{q})}{\tilde{q}} \frac{1-p}{p} P(G) \right] \\ &= \frac{\bar{q}}{\tilde{q}} p \frac{1-p}{p} \frac{n}{R} \binom{n-1}{R-1} (1-p)^{n-R+1} p^{R-1} \left[\tilde{q} P(I) \left(\frac{1-p}{p} \right)^{2R-n-2} - (1-\tilde{q}) P(G) \right] \\ &= \frac{\bar{q}}{\tilde{q}} (1-p) \frac{n}{R} [\Pi_{\tilde{q}}(R, n) - \Pi_{\tilde{q}}(R-1, n-1)]. \end{aligned}$$

We are interested in whether the quantity $\frac{\bar{q}}{\tilde{q}} (1-p) \frac{n}{R}$ exceeds 1. As c approaches zero, the optimal jury size grows and n^*/R^* approaches 2. We now show that $\frac{\bar{q}}{\tilde{q}} (1-p)2 > 1$ if and only if $\bar{q} < 1/2$.

The statement $\bar{q}(1-p)2 > \tilde{q}$ is equivalent to the statement

$$\frac{(1-\tilde{q})}{\tilde{q}} > \frac{(1-\bar{q})(1-p)2}{\bar{q}(1-p)2}$$

which, in light of equation (A.11), is equivalent to

$$\frac{(1-\bar{q})(1-p)2}{\bar{q}(1-p)2} \frac{1-p}{p} < \frac{(1-\bar{q})}{\bar{q}}.$$

With some algebra, this inequality is shown to be equivalent to $\bar{q} < \frac{1}{2}$.

This shows that when $\bar{q} < \frac{1}{2}$ then $\Pi_{\tilde{q}}(R, n) - \Pi_{\tilde{q}}(R-1, n-1) < \Pi_{\bar{q}}(R+1, n+1) - \Pi_{\bar{q}}(R, n)$. This shows that the maximal possible value of the marginal incentive to stay informed in a jury of size n and rule R , which is realized at

\bar{q} , is dominated by the incentive to stay informed in a jury of size $n + 1$ and rule $R + 1$ at \bar{q} . Consequently, given any jury of size n with a cost of error within $[\underline{q}, \bar{q}]$, there is another jury of larger size and cost of error closer to $1/2$ in which jurors have greater incentives to acquire information.

Finally, we must ensure that, given any $\hat{q} < 1/2$, the corresponding \bar{q} is also smaller than $1/2$. To this end, notice that $\bar{q} < 1/2$ is equivalent to $(1 - \bar{q})/\bar{q} > 1/2$; then, observing that

$$\frac{(1 - \bar{q})}{\bar{q}} > \frac{(1 - \hat{q})}{\hat{q}} \left(\frac{1 - p}{p} \right)^2,$$

we get that a sufficient condition for $\bar{q} < 1/2$ is that $\frac{(1 - \hat{q})}{\hat{q}} \left(\frac{1 - p}{p} \right)^2 > 1/2$. This justifies the p -far condition in the statement of the proposition.

We obtain the implication for the case in which $\bar{q} > 1/2$ by repeating the same steps as before, but with q being replaced with $(1 - q)$. \parallel

Proof of Proposition 4

Proof. Fix $P(I)$, and let R, n denote the optimal rule and jury size when the accuracy is \hat{p} . We know that $R = R_{\hat{p}}^S(n)$, and we denote with \underline{p} and \bar{p} the lowest and highest value of p such that $R_{\hat{p}}^S(n) = R$. By definition, $\hat{p} \in [\underline{p}, \bar{p}]$. Without loss of generality, assume $qP(I) > (1 - q)P(G)$ (otherwise, switch the notation). Also, to fix ideas assume that n is even (the analysis for n odd is similar).

We wish to characterize the marginal incentive to stay informed in a jury of size n as p decreases within $[\underline{p}, \bar{p}]$. To this end, we notice that $R_{\hat{p}}^S(n - 1)$ goes from $R - 1$ to R as p decreases within $[\underline{p}, \bar{p}]$. Therefore, if we denote with \tilde{p} the value at which $R_{\hat{p}}^S(n - 1)$ switches to R , the marginal incentive to stay informed in a jury of size n is

$$\begin{cases} \Pi_p(R, n) - \Pi_p(R - 1, n - 1) & \text{for } \tilde{p} \leq p < \bar{p} \\ \Pi_p(R, n) - \Pi_p(R, n - 1) & \text{for } \underline{p} \leq p < \tilde{p}. \end{cases}$$

Recall that

$$\begin{aligned} & \Pi_p(R, n) - \Pi_p(R - 1, n - 1) \\ &= \binom{n - 1}{R - 1} (1 - p)^{n - R + 1} p^{R - 1} \left[qP(I) \left(\frac{1 - p}{p} \right)^{2R - n - 2} - (1 - q)P(G) \right]. \end{aligned}$$

The term inside brackets is decreasing in p since R is strictly greater than $n/2$ (if n were equal or smaller to $n/2$ then we would have $\Pi_p(R, n) - \Pi_p(R, n - 1) < 0$, so not all jurors would become informed, contradicting the optimality of R). The term outside brackets is decreasing in p if $R - 1 < np$, which is guaranteed by Theorem 1 for n sufficiently large. Thus, for $\tilde{p} \leq p < \bar{p}$ the marginal incentive to stay informed is decreasing in p when n is large enough. In fact, the above quantity assumes value zero at \bar{p} . For $\underline{p} \leq p < \tilde{p}$ the marginal incentive to stay informed is

$$\begin{aligned} & \Pi_p(R, n) - \Pi_p(R, n - 1) \\ &= \binom{n - 1}{R - 1} (1 - p)^{n - R} p^R \left[(1 - q)P(G) - qP(I) \left(\frac{1 - p}{p} \right)^{2R - n} \right]. \end{aligned}$$

Although this expression is not always increasing in p , it assumes value zero at \underline{p} and it crosses the curve given by $\Pi_p(R, n) - \Pi_p(R - 1, n - 1)$ only once (at \tilde{p}). This shows that the marginal incentive to stay informed in a jury of size n is not monotonic on $[\underline{p}, \bar{p}]$, and that it is decreasing on $[\tilde{p}, \bar{p}]$.

Let us first prove the second part of Proposition 4, namely, that there is a set \mathfrak{A} of accuracies which are smaller than \hat{p} , such that the optimal committee size for any $p' \in \mathfrak{A}$ is strictly larger than for \hat{p} . We break this proof into two parts, depending on whether $\hat{p} \in [\tilde{p}, \bar{p}]$ or $\hat{p} \in [\underline{p}, \tilde{p}]$. We start by treating the case $\hat{p} \in [\tilde{p}, \bar{p}]$. We show that there is an accuracy level smaller than \tilde{p} and a jury size strictly larger than n , at which the marginal incentives to stay informed are larger than the marginal incentive to stay informed in a jury of size n and accuracy \tilde{p} . Since the marginal incentives to stay informed is decreasing on $[\tilde{p}, \bar{p}]$, this proves the second part of Proposition for the case $\hat{p} \in [\tilde{p}, \bar{p}]$. Consider the marginal incentive to stay informed in a jury of size $n + 1$ and accuracy \underline{p}

$$\begin{aligned} & \Pi_{\underline{p}}(R + 1, n + 1) - \Pi_{\underline{p}}(R, n) \\ &= \binom{n}{R} (1 - \underline{p})^{n - R + 1} \underline{p}^R \left[qP(I) \left(\frac{1 - \underline{p}}{\underline{p}} \right)^{2R - n - 1} - (1 - q)P(G) \right]. \end{aligned} \quad (\text{A.12})$$

By definition of \underline{p} we have

$$\left(\frac{1-\underline{p}}{\underline{p}}\right)^{2R-n} = \frac{(1-q)P(G)}{qP(I)}.$$

Substituting into (A.12) yields

$$\begin{aligned} & \Pi_{\underline{p}}(R+1, n+1) - \Pi_{\underline{p}}(R, n) \\ &= \binom{n}{R} (1-\underline{p})^{n-R+1} \underline{p}^R (1-q)P(G) \left[\left(\frac{1-\underline{p}}{\underline{p}}\right)^{-1} - 1 \right] \\ &= \frac{n}{R} \binom{n-1}{R-1} (1-\underline{p})^{n-R+1} \underline{p}^R (1-q)P(G) \left[\frac{2\underline{p}-1}{1-\underline{p}} \right]. \end{aligned} \quad (\text{A.13})$$

Similarly,

$$\begin{aligned} & \Pi_{\tilde{p}}(R, n) - \Pi_{\tilde{p}}(R-1, n-1) \\ &= \binom{n-1}{R-1} (1-\tilde{p})^{n-R+1} \tilde{p}^{R-1} \left[qP(I) \left(\frac{1-\tilde{p}}{\tilde{p}}\right)^{2R-n-1} - (1-q)P(G) \right]. \end{aligned}$$

Since by definition of \tilde{p} we have

$$\left(\frac{1-\tilde{p}}{\tilde{p}}\right)^{2R-n-1} = \frac{(1-q)P(G)}{qP(I)},$$

the same steps as above lead to

$$\begin{aligned} & \Pi_{\tilde{p}}(R, n) - \Pi_{\tilde{p}}(R-1, n-1) \\ &= \binom{n-1}{R-1} (1-\tilde{p})^{n-R+1} \tilde{p}^{R-1} (1-q)P(G) \left[\frac{2\tilde{p}-1}{1-\tilde{p}} \right]. \end{aligned} \quad (\text{A.14})$$

We wish to show that (A.13) is greater than (A.14). Since $\tilde{p}n/R$ is greater than 1 for n sufficiently large (which is guaranteed by our assumption that c is small), the desired inequality is implied by

$$(1-\underline{p})^{n-R+1} \underline{p}^R \left[\frac{2\underline{p}-1}{1-\underline{p}} \right] > (1-\tilde{p})^{n-R+1} \tilde{p}^R \left[\frac{2\tilde{p}-1}{1-\tilde{p}} \right].$$

This inequality is verified if the function $(1-p)^{n-R} p^R (2p-1)$ is decreasing in p . Taking logarithms, we wish to show that

$$\frac{\partial}{\partial p} [(n-R) \log(1-p) + R \log(p) + \log(2p-1)] = -\frac{(n-R)}{1-p} + \frac{R}{p} + \frac{2}{2p-1} < 0.$$

Rearranging the inequality yields

$$\frac{R}{n} < p - \frac{1}{n} \frac{2p(1-p)}{2p-1}.$$

As n becomes large, the R.H.S. approaches p , and the inequality is verified. This shows that the incentives to stay informed are larger in a jury with $n+1$ members and accuracy \underline{p} than in a jury with n members and accuracy \tilde{p} (and hence any p in (\tilde{p}, \bar{p})).

We now deal with the case $\hat{p} \in [\underline{p}, \tilde{p}]$. When \hat{p} is in this range, the marginal incentive to stay informed is $\Pi_{\hat{p}}(R, n) - \Pi_{\hat{p}}(R, n-1)$, which we now show to be smaller than $\Pi_{\underline{p}}(R+1, n+1) - \Pi_{\underline{p}}(R, n)$, the marginal incentive to stay informed in a jury of size $n+1$ at \underline{p} . Substitute from (A.9) and (A.12) to obtain the mathematical expression that needs to be true for large n ,

$$\begin{aligned} & \binom{n-1}{R-1} (1-\hat{p})^{n-R} \hat{p}^R \left[(1-q)P(G) - qP(I) \left(\frac{1-\hat{p}}{\hat{p}}\right)^{2R-n} \right] \\ & < \binom{n}{R} (1-\underline{p})^{n-R+1} \underline{p}^R \left[qP(I) \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2R-n-1} - (1-q)P(G) \right]. \end{aligned} \quad (\text{A.15})$$

We will show that as n grows the R.H.S. of the above equation goes to zero more slowly than the L.H.S. First, note that in the above expression R is a function of n and grows at rate $n/2$. Thus, the terms in brackets do not change as we make n

grow by increments of 2 (so as to keep n even). The right-hand binomial term grows at exactly the speed of the left-hand binomial term, since their ratio equals n/R which converges to a constant as n grows. Finally, the ratio

$$\frac{(1 - \underline{p})^{n-R+1} \underline{p}^R}{(1 - \hat{p})^{n-R} \hat{p}^R}$$

behaves, for large n , as

$$\left[\frac{(1 - \underline{p}) \underline{p}}{(1 - \hat{p}) \hat{p}} \right]^{n/2}$$

since R grows at rate $n/2$. The ratio in brackets is greater than 1 because by assumption $0.5 \leq \underline{p} < \hat{p}$, so the above expression will go to infinity as n grows. This shows that the inequality in (A.15) is verified for large n , which concludes the proof of the second part of Proposition 4.

Now, let us prove the first part of Proposition 4, namely, that given any $p < \hat{p} < 1/2$ the optimal committee size is larger or equal at p than at \hat{p} for large n . We will show that this is the case for any $p \in [\underline{p}, \bar{p}]$; it is clear how to extend the argument to the general case in which $p < \underline{p}$.

The case $p = \underline{p}$ has been treated above, since we have shown that at that accuracy level the incentives to stay informed in a committee of size $n + 1$ are larger than in any committee with accuracy \hat{p} and size n . The case $p \in [\bar{p}, \bar{p}]$ has also been treated above, since we have shown that in that range the incentive to stay informed in a committee of size n are decreasing in p . The only case that remains is the case $p \in [\underline{p}, \bar{p}]$. We now show that the incentive to stay informed are decreasing in p for n sufficiently large. This will show that choosing p slightly smaller cannot decrease the optimal committee size, and will conclude the proof of the theorem. For $\underline{p} \leq p < \bar{p}$ the marginal incentive to stay informed is

$$\Pi_p(R, n) - \Pi_p(R, n - 1) = \binom{n-1}{R-1} (1-p)^{n-R} p^R \left[(1-q)P(G) - qP(I) \left(\frac{1-p}{p} \right)^{2R-n} \right].$$

We want to show that for large n this expression is decreasing in p . Take logarithms of the expression and notice that the term in brackets is independent of n (since $R = R^S(n)$ grows at a speed of exactly $n/2$). Thus, the derivative with respect to p of the log of the term in bracket does not change with n . We now show that the derivative of the log of the term outside brackets,

$$\log(1-p)^{n-R} + \log(p)^R,$$

is a negative number that converges to negative infinity. Differentiating with respect to p yields

$$-\frac{(n-R)}{(1-p)} + \frac{R}{p} = \frac{R}{(1-p)} \left\{ -\frac{n}{R} + \frac{1}{p} \right\}.$$

As n becomes large the term in curly brackets converges to the negative number $-2 + (1/p)$, while the term outside brackets goes to infinity. This large negative term will dominate the derivative of the marginal incentive to stay informed. Thus, for large n the marginal incentive to stay informed is decreasing in p , as was to be shown. \parallel

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