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Economic Theory

Research Article

Campaign spending regulation in a model of redistributive politics★

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Summary. We consider a model in which parties that differ in perceived valence choose how to allocate electoral promises (money, pork-barrel projects) among voters. The party perceived to be less valent has a greater incentive to "sell out" to a favored minority and completely expropriate a fraction of the electorate. By reducing the difference in perceived valence, campaign-finance regulations may reduce the extent of the expropriation and achieve a more equitable political outcome. We analyze various instruments of campaign-finance regulation from this perspective.

Keywords and Phrases: Campaign spending regulation, Redistributive politics.

JEL Classification Numbers: D72, H2.

1 Introduction

This paper is concerned with the effects of campaign spending and its regulation within a model of redistributive politics. The model is one in which parties (or candidates) compete for votes by promising slices of a cake. This is a benchmark model in much of the literature in political economics and political science. Most treatments of this model select as solutions allocations that are not egalitarian—not all voters receive equal slices. The tendency towards unequal allocations was noted by Shubik [28] and is the focus, for example, of Myerson [25] and Laslier and Picard [19]. Inequality among voters is viewed as an undesirable property of the solution.

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In this paper, we solve for a variant of this benchmark model in which parties are asymmetric. We are interested in the inequality of the solution as a function of the asymmetry between parties.

In our model, parties (or their leaders) are perceived by voters as differing in valence. A party with greater valence is believed by voters to be able to generate more resources (a bigger cake) once in office. That party will therefore be able to make electoral promises under a less stringent budget constraint, and so has an advantage.[1] In this environment, we show that electoral imbalance translates into fiscal inequality. To see why, consider the position of a party that is perceived as less valent. In order to win a voter, that party must offer that voter at least as much as the competing party. However, the less valent party is subject to a more stringent budget constraint and, in equilibrium, chooses not to treat all voters equally. Instead, the less valent party ignores a fraction of the electorate in order to concentrate its resources on the remaining voters. In this sense, the disadvantaged party "sells out" to a favored sub-group of the electorate and completely expropriates the remaining fraction of the electorate. The greater the disadvantage at the electoral stage, the larger the fraction of voters that are expropriated in the disadvantaged party's platform, and the greater the degree of fiscal inequality.[2]

Having argued that the degree of fiscal inequality is related to the difference in the voters' perception of parties, we set out to evaluate the role that campaign spending regulation can play in reducing the asymmetry between parties. This exercise is similar in spirit to that of Myerson [25]. Myerson [25] asks a constitutional design question by comparing electoral systems in terms of the inequality that they induce in equilibrium. We take the electoral systems as given, and compare campaign spending regulations according to the same criterion.

In our model, the campaign spending stage precedes electoral competition. In the campaign spending stage parties start out with different perceived valences, and can add to their valence by spending campaign funds. In effect, parties in the campaign spending stage are engaged in a spending contest for advantage in the electoral stage. Let us first concentrate on the incentives for parties to spend campaign funds in the absence of regulation. A fundamental feature of the vote shares (as determined in equilibrium at the electoral stage of our model) is this: if campaign expenditure is increased by the same amount for both parties, then the underdog gains in vote share.[3] Thus, if a sufficiently large amount is added to both parties' expenditures, any initial differential in perceived valence becomes negligible in the electoral game, which makes sense since the ratio of perceived

[1] Valence can be interpreted as competence in administrative tasks, ability to make efficient government decisions, abstinence from corruption, and generally the practice of those virtues that minimize the deadweight loss involved in government redistribution. An alternative interpretation is to imagine that both parties have the same budget constraint but the promises of one party are discounted, due to lower popularity or name recognition.

[2] In our setup the disadvantaged party's platform has an effect on policy even though that party receives less than 50 percent of the votes. We discuss this point in Section 2.1. In Section 6 we state precisely what we mean by inequality.

[3] To be precise, in our model this property holds if both parties start from the same amount of advertising. This is all that is required for our argument to go through, as shown in Section 4.

valences converges to 1.[4] This "catching-up effect" is a natural property if we take the position, as we do in this paper, that campaign expenditure increases (or is a substitute for) a party's perceived valence. Note that this property implies that the productivity of campaign funds, measured in terms of the electoral advantage afforded by a unit of expenditure, must be greater for the disadvantaged party. But then the incentives to spend campaign funds must be larger, caeteris paribus, for the disadvantaged party. In our model this effect will lead the disadvantaged party to outspend its opponent in the attempt to catch up.[5]

This argument suggests that, since in our model campaign spending reduces the gap in perceived valence, a laissez-faire approach to campaign spending might be beneficial. In fact, in the special case in which the effect of spending on perceived valence is linear, we show that under laissez faire the disadvantaged party completely catches up and a perfectly balanced outcome is achieved. In such circumstances, campaign finance regulation can only hinder the catching-up process. Similarly, in the more general case in which laissez faire does not achieve perfect balance we show that regulations that provide free in-kind transfers in equal amounts to both parties have anti-competitive effects, because they make it harder for the disadvantaged party to catch up. The effect of a regulation setting a cap on expenditures is less immediate; in equilibrium, imposing a cap decreases the expenditure of both parties due to a strategic effect on the expenditure of the ex ante favored party. Nevertheless, we are able to show that the impact of a cap is stronger on the spending of the ex ante disadvantaged party, and so imposing a cap reduces the equilibrium vote share of that party. Again contrary to conventional wisdom, spending caps are anti-competitive. The effects of per-seat reimbursements and of matching funds on campaign spending are more complicated. Under these types of schemes, financing becomes cheaper for *both* parties. Thus, both parties increase their campaign outlays, and the total effect on vote shares depends on which party increases its expenditure more. In our model, that depends on the productivity of spending; we identify tight conditions on the curvature of the function that transforms expenditures into perceived valence, under which per-vote reimbursements and matching funds are pro- or anti-competitive.

1.1 Related literature

Asymmetric electoral competition

Our analysis contributes to a recent literature on electoral competition between asymmetric candidates or parties (see e.g. Aragones and Palfrey [2] and Groseclose [16]). One point of difference with much of this literature is that we assume that parties maximize their vote share instead of the probability of winning. This assumption makes our model a closer representation of electoral systems with proportional representation.

[4] See Jacobson [18] for evidence that campaing spending is more productive for the challenger than for the incumbent in US House elections.

[5] We assume that both parties have the same opportunity cost of funds. More on this in Section 2.3.

Redistributive politics

The literature on redistributive models of electoral competition is surveyed by Dixit and Londregan [13]. Our model of redistributive politics extends Myerson [25] to the case of asymmetric parties. An important feature of the asymmetric model is that in equilibrium voters are treated differently by different parties, and the difference in the parties' electoral appeal translates into polarized fiscal policies. Laslier and Picard [19] analyze a version of Myerson's model in which there are a finite number of voters and two parties and show that when the number of voters converges to infinity the inequality is smaller than in Myerson's model.

We endogenize the electoral appeal of parties through the campaign financing activity, and so obtain a model in which the extent of fiscal inequality is endogenous. In this sense, our analysis complements Dixit and Londregan [13], who call attention to the importance of fiscal inequality. In their model, inequality in the treatment of voters results from two features. One is the responsiveness of different voters to electoral promises: groups of voters which are more responsive receive more generous electoral promises. The second feature is the presence of a concern for inequality into the payoff functions of agents (parties and voters). In contrast, in our stylized model voters are identical and no political actor has a concern for equity. In our model, fiscal inequality emerges in equilibrium as the strategy of the party with less electoral appeal.

Campaign spending

The formal literature on campaign spending is vast; here we discuss the work to which this paper is more closely related, and we refer to Morton and Cameron [23] for a general survey. Most papers concentrate on implications for the probability of winning of incumbent vs. challenger candidates (Levitt [20], Erikson and Palfrey [14]) or on the amount of campaign contributions (Baye, Kovenock, and DeVries [6], Che and Gale [9]). In order to focus on campaign spending, most of this literature models the behavior of voters through an exogenous "vote production function" whose inputs are campaign contributions. In contrast, because we explicitly model the stage of electoral competition, we are able to draw implications for policy outcomes.

The effect of campaign contributions on policy outcomes is explicitly modeled in the literature on lobbying. In this literature, inefficiencies result from politicians pandering to lobbies, which are groups with preferences different from the median in the population. In most of the lobbying literature electoral competition remains in the background as lobbying is directed at an incumbent politician; an exception is Baron [5], who studies a lobbying model in which two parties compete for office. One key difference between our model of campaign spending and lobbying models is in the policy implications. In these models, the inefficiency is a result of lobbying, and so the best policy is to limit campaign financing. Some recent papers have analyzed models in which campaign advertising has a social benefit. Prat [27] analyzes the trade-off between the informational benefits of advertising and its social costs. In his analysis, the competence of candidates is not perfectly known. An interest group, which is better informed than the voters, gives contributions to candidates. Since, the group offers more contributions to a candidate he believes to be more competent, and thus more likely to be elected, contributions play the role of a signal. Limiting contributions and advertising has ambiguous welfare effects: the losses in terms of information about competence are to be balanced with policy distortions. Coate [10,11] analyzes models with directly informative advertising. In Coate [11], voters are uninformed about candidates' ideologies. He shows that when contributions come from interest groups who try to get their favorite candidate elected, limits on advertising redistribute welfare from moderate voters to members of the interest groups. In Coate [10], voters are uninformed about the competence of candidates. He shows that when advertising is financed by contributions from interests groups in exchanges for favors, the informational value of advertising on competence is mitigated by the knowledge that advertising is linked with the inefficient distribution of favors. In this context, a ban on advertising may well be Pareto improving.

In our model, instead, we take no stand on the origin of funds. We focus on the consequences of advertising expenditures in the context of an asymmetric electoral competition in which one party is initially disadvantaged. Forbidding campaign advertising has a negative impact since it maximizes the difference in electoral appeal and hence the incentive of the disadvantaged party to run on a fiscally polarized platform.

1.2 Outline of the paper

Section 2 presents the model and the timing of events. Section 3 solves for the equilibrium of the electoral game. Section 4 analyzes the campaign spending stage. Section 5 discusses the effects of campaign spending regulations. We discuss the fiscal inequality generated by unbalanced elections in Section 6. Section 7 extends the analysis to three or more parties. We show how the asymmetry between parties translates into equilibrium promises. Section 8 concludes.

2 Model

The model more closely portrays proportional systems rather than majoritarian systems. This is because of two features of the model. One is that parties maximize their vote share, not the probability of their vote share exceeding 50 percent. The second feature is that the implemented policy is not necessarily the platform of the party whose vote share exceeds 50 percent. The implemented policy also partly reflects the platform of the minority party. Albeit stylized, these two features are a consistent with Lijphart's [22] description of consensus (as opposed to majoritarian) political systems. Lijphart [22] identifies an archetypal model of *consensual democracy* defined by the following institutional features. A proportional representation system of elections; many parties; broad coalitional governments in which most of the important parties share governments; a strong bicameralism; balance between executive and legislative power; a high level of delegation of power to local authorities.[6]

In one important aspect our model departs from Lijphart's description of consensus democracies: it only has two parties. This is done to simplify the analysis of the campaign spending game, which would is quite involved with more than two parties. In Section 7, we extend the model to analyze competition between more than two parties.

2.1 Electoral stage

Agents

We extend the model of redistributive politics of Myerson [25] to the case of elections with a favored party. There are two parties, numbered 1 and 2, which compete in an election. Before the election, parties make binding promises to each voter concerning the allocation of a budget if elected.

To avoid the complexity of a large finite number of voters, we will specify the number of voters to be infinite: there is a continuum of measure 1 of voters. V denotes the set of voters, and v a voter.

Strategies

A party can divide a budget among voters. Voters believe that Party i can allocate a total budget of $1 + \alpha_i$ (which amounts to $1 + \alpha_i$ per voter).[7] The parameter α_i summarizes the voters' perception of Party i's competence. We refer to α_i as to the *perceived valence* of Party i. The values of α_i are determined in a campaign spending game prior to the election, and in the election game voters and parties take the values of α_i as given.

Note that we do not need to take a stand here on the actual size of α_1 and α_2: only the *beliefs* of voters need to be specified to be able to solve for the equilibrium behavior of parties and voters. In Section 6, we address the question of how to take into account the size of the budget to make meaningful comparisons.

Policy implementation

To derive voting behavior we need to describe how vote shares translate into policy. We wish to capture an important feature of parliamentary systems based on proportional representation: power sharing. With this term we indicate the fact that the party with the highest vote share does not generally wield all the power (even though influence tends to be less important for smaller parties.) In Belgium, Israel, and Italy, for example, coalition governments are the norm, and parties with less than a plurality of the votes participate in government and influence policy. In these

[6] Switzerland and Belgium are closest to this archetype.

[7] One way to think of the budget is to imagine that it represents the proceeds of taxation. We can think of each voter being endowed with one unit of money. Once in office, Party i taxes all voters' endowment, invests the proceeds at interest rate α_i, and then redistributes the principal and interest.

countries, parties that are not in government are sometimes enlisted to support government policies "from the outside," in exchange for influence on other dimensions of policy.

This richness of behavior cannot be fully captured in our simple 2-party model. However, the notion of power-sharing can be approximated by allowing the policy outcome to reflect the parties's platforms in proportion to their vote share. There are various models of policy formation in which policy reflects the parties's platforms in proportion to their vote share. The first model is one in which Party i gets a fraction S_i of the seats in parliament, where S_i denotes Party i's vote share. Then, a formateur is chosen at random among the parliamentary representatives and gets to form government. The implemented policy is the platform of the formateur's party. We refer to this procedure of policy implementation as *probabilistic compromise*, reflecting the fact that the platform of one of the two parties becomes policy with a probability that equals their vote shares. The device of probabilistic compromise was used by, among others, Grossman and Helpman [17]. For an analysis of various abstract models of probabilistic compromise see Fishburn and Gerlein [15].

The second possible implementation of power-sharing mimics a compromise achieved by *splitting the spoils of office*. A "government" is formed that includes both parties. Party i has a bargaining power within government of S_i, and is allocated a share S_i of the budget to distribute among voters. Parties distribute the fraction of the budget that they are allocated according to the promises in the voting game.[8]

These two assumptions on policy implementation are equivalent in the sense that both give rise to the same voting behavior: a voter votes for the party which promises him/her the greatest expected transfer. The only place in this paper in which the difference between the two is of any consequence is Section 6, where we adopt probabilistic compromise on grounds of mathematical tractability.

Voting behavior

Given any of the two assumptions on policy implementation, it is optimal for a voter to vote for the party which promises him/her the greatest expected transfer.[9] Let $x_i(v) \in [0, \infty)$ denote the fraction of Party i's budget promised to voter v, so that $\int_V x_i(v)dv = 1$. A voter who receives promises of x_1, x_2 votes for Party 1 if $(1 + \alpha_2)x_2 < (1 + \alpha_1)x_1$. Thus if $0 \leq \alpha_2 < \alpha_1$, Party 1 is favored over Party 2.

2.2 Campaign spending stage

Before parties enter the electoral stage we assume that they can spend campaign funds to increase their perceived valence. Party i is endowed with an initial level of perceived valence $\underline{\alpha}_i$. We assume $\underline{\alpha}_1 > \underline{\alpha}_2$; thus, Party 1 has larger initial perceived valence.

[8] This interpretation is found in Myerson [25]. Note that in our model voters believe that party i will increase its share of the budget by a factor α_i before allocating it.

[9] To be precise, this behavior is optimal for a voter when there are a large number (but finite) of voters, so that there is a positive probability that a vote affects the outcome. Although we assume a continuum of voters, we view the continuum as an approximation of a large but finite number of voters.

Parties can add to their initial valence through campaign expenditure. By spending m_i Party i increases its perceived valence by $h(m_i)$, resulting in a final perceived valence of

$$\alpha_i \stackrel{\text{def}}{=} \underline{\alpha}_i + h\left(m_i\right).$$

We assume that $h(\cdot)$ is increasing, concave, twice differentiable, and that $h(0) = 0$. We also assume $h'(0) = \infty$; this assumption ensures that in equilibrium both parties spend some campaign funds.

Because we assume that spending funds necessarily increases a party's perceived valence, our model is one of *persuasive campaigning* (see Stokes [30], Baron [5], Mueller and Stratmann [24], Stromberg [31]). In order to focus on the effects of campaign expenditure on redistribution, we take a "black-box" approach to the question of why campaign spending works. It would be desirable to open the black box and understand the rational mechanism for why voters respond to campaign spending. This is the goal of models in which spending increases the level of the voters' information, either directly (Austen-Smith [3], Coate [11]) or indirectly (Potters, Sloof, and van Winden [26], Prat [27]).

2.3 The objective of parties

Parties maximize their vote share minus the opportunity cost of funds,

$$S_i - tm_i.$$

Here, S_i denotes the vote share obtained by Party i, and t is the marginal opportunity cost of funds. Note there is no loss of generality in assuming that the cost of funds be linear as opposed to convex, for example.[10] Once we have chosen the linear normalization, however, we must take a stand on how parties perceive the vote share and thus, implicitly, on the trade-off between vote shares and funds. We assume each party maximizes its vote share, as opposed for example to maximizing the probability of obtaining a majority of the votes which would be a nonlinear function of the vote share.[11]

Another important feature of the parties' objective function is that both parties are assumed to have the same value of vote shares and the same opportunity cost of funds. We return to this issue in the Conclusions.

2.4 Timing

There are three stages to the game. We henceforth refer to stage 1 as the *campaign spending game*, and to the game in stages 2 and 3 as the *electoral game*.

[10] A convex cost function would be equivalent to assuming a more concave advertising production function.

[11] See Snyder [29] for another paper in which two parties maximize vote share instead of the probability of winning.

1 (Campaign Spending Stage). The parties simultaneously choose m_i, $i = 1, 2$.

2 (Electoral Stage). After observing the outcome of the previous stage, parties make promises to voters simultaneously and independently.

3 (Voting). Each voter v gets promises $x_1(v), x_2(v)$ from parties. After observing the offers, voter v votes for Party i if

$$(1 + \underline{\alpha}_i + h(m_i)) \cdot x_i(v) > (1 + \underline{\alpha}_j + h(m_j)) \cdot x_j(v).$$

If equality holds, the voter randomizes with equal probability.

3 The electoral game (Stages 2 and 3)

In this section we solve for the equilibrium of the electoral game. We take the values α_1 and α_2 as given and assume that $\alpha_1 \geq \alpha_2$.

A pure strategy in the electoral game specifies a promise of a fraction of the budget to each voter. Formally, a pure strategy is a function $x : V \to [0, +\infty)$ that satisfies the balanced budget condition $\int_V x(v)dv = 1$. It is well-known that there is no equilibrium in pure strategies.

To characterize the equilibrium in mixed strategies, we follow the approach of Myerson [25]. Myerson focusses on mixed strategy equilibria in which each voter receives a promise from Party j which is an independent draw of a random variable with c.d.f. F_j. $F_j(x)$ denotes the fraction of the voters to whom candidate j will offer less than x. From now on we refer to F_j as a budget allocation. Since offers to different voters are independent realizations of a random variable, these mixed strategies generate strategic uncertainty about how much each particular voter is promised by Party j. The uncertainty thus created eliminates the cherry-picking opportunities for Party i and creates the conditions for existence of a Nash equilibrium.[12] We now solve for the Nash Equilibrium of the electoral game.

Since Party i draws promises to voters from $F_i(x)$, the budget constraint reads $\int_0^\infty x dF_i(x) = 1$. If a voter is promised a fraction x of Party i's budget, the probability that she votes for i is $F_j(\frac{1+\alpha_i}{1+\alpha_j}x)$. Party i maximizes vote share by solving

$$\max_{F_i} \int_0^\infty F_j\left(\frac{1+\alpha_i}{1+\alpha_j}x\right) dF_i(x) \text{ subject to } \int_0^\infty x dF_i(x) = 1. \qquad (1)$$

Definition 1 *Assume* $\alpha_1 \geq \alpha_2$. *We call* $\rho = (1 + \alpha_2)/(1 + \alpha_1)$ *the perceived valence ratio (PVR).*

[12] Assuming as we do that draws are independent poses a technical problem because we have a continuum of draws, one for each voter. We refer to Alos-Ferrer [1] for an overview of mathematical solutions that have been devised to overcome the technical problem. In our paper, an easy way to get around the problem would be to relax the continuum assumption and assume instead that the number of voters is finite (presumably large). All the analysis would then carry through unchanged, provided that we require the parties' budget constraint to be satisfied in expectation. The drawback of this alternative modeling assumption is that the parties' budget constraint would be satisfied in expectation only, though this drawback becomes negligible when the number of voters becomes large.

The PVR is between zero and one, with values close to zero indicating a large perceived valence differential and a lopsided electoral contest. The PVR completely determines the equilibrium of the electoral stage.

The next theorem solves for the equilibrium of the electoral stage. To understand how the vote shares arise, observe that in order to match the perceived value of a given promise from its opponent, the disadvantaged party must allocate a greater fraction of its budget. It follows that the disadvantaged party cannot compete for all voters on equal terms with its opponent. In equilibrium, the disadvantaged party will choose to concentrate its promises on a fraction ρ of the electorate. For that fraction of the electorate, both parties compete with the same *perceived* intensity, and split the votes equally. The remaining $1-\rho$ of the electorate is ignored by the disadvantaged party and votes for the advantaged party.

Theorem 1 *Assume $\alpha_1 \geq \alpha_2$. In the unique equilibrium, Party 2 promises zero to a fraction $1-\rho$ of the electorate (randomly chosen) and allocates its budget to the remaining voters according to a uniform distribution on $[0, 2/\rho]$. Party 1's budget promises (to all voters) are distributed according to a uniform distribution on $[0, 2]$.*

The vote share of Party 2 is $S_2 = \rho/2$. The vote share of Party 1 is $S_1 = 1-(\rho/2)$.

Proof. Denote by F_1^* and F_2^* the candidate equilibrium presented above. Thus, $F_1^*(z)$ is a uniform distribution on $[0, 2]$ and $F_2^*(z) = (1-\rho)+\rho(z\rho/2)$ on $[0, 2/\rho]$. The c.d.f.'s F_1^* and F_2^* satisfy the budget constraint, since both have an expected value of 1. Let us verify that F_1^* is a best response to F_2^*. Since Party 2 promises zero to a positive fraction of voters, Party 1's best response must entail $F_1^*(0) = 0$, since it could never be rational for Party 1 to risk tying for a voter. Having ruled out ties, Party 1's vote share for any feasible F_1 reads, using expression (1),

$$
\begin{aligned}
&\int_0^{\infty} F_2^*\left(\frac{x}{\rho}\right) dF_1(x) \\
&\leq \int_0^{\infty} (1-\rho) + \frac{x}{\rho}\left(\frac{\rho^2}{2}\right) dF_1(x) \\
&= (1-\rho) + \left(\frac{\rho}{2}\right)\int_0^{\infty} x dF_1(x) \\
&= (1-\rho) + \left(\frac{\rho}{2}\right),
\end{aligned}
$$

where the last equality follows from feasibility of F_1. The inequality will hold with equality whenever the support of F_1 is contained in $[0, 2]$, which is the case for the candidate equilibrium F_1^*. Thus, F_1^* achieves $1-\frac{\rho}{2}$, the maximal payoff among all feasible F_1's. Therefore, F_1^* is a best response to F_2^*.

A very similar argument establishes the optimality of F_2^* against F_1^*. The equilibrium of the electoral game is also unique in the class of equilibria in budget allocations; this is proved in the Appendix by exploiting the relationship between the electoral game and the all-pay auction model with complete information, for which uniqueness results have been developed by Baye, Kovenock, and DeVries [7]. □

Theorem 1 shows that the equilibrium distribution of utility from promises, *as perceived by voters*, has the same support for both parties; for example, if a voter were to receive by each party the highest promise ever made by that party, that voter would be indifferent as to which party to vote for. Another implication is that some voters are completely expropriated under the disadvantaged party's platform. We focus on this aspect of the model in Section 6.

4 The campaign spending game (Stage 1)

In this section we solve for the Nash Equilibrium of the campaign spending stage. The primitives in this section are the equilibrium vote shares derived in Section 3. We begin our analysis under the assumption that $\alpha_1 \geq \alpha_2$, that is, Party 2 does not completely overcome its initial handicap; we verify in Theorem 3 that indeed this is the case.

Denote by $S_i(m_1, m_2)$ the vote shares in an electoral subgame where parties have spent m_1 and m_2:

$$S_1\left(m_1, m_2\right) \stackrel{\text{def}}{=} 1 - \frac{1}{2}\rho\left(m_1, m_2\right)$$
$$S_2\left(m_1, m_2\right) \stackrel{\text{def}}{=} \frac{1}{2}\rho\left(m_1, m_2\right),$$

where

$$\rho\left(m_1, m_2\right) \stackrel{\text{def}}{=} \frac{1 + \underline{\alpha}_2 + h\left(m_2\right)}{1 + \underline{\alpha}_1 + h\left(m_1\right)}. \tag{2}$$

Party i chooses its expenditure to solve

$$\max_{m_i} S_i\left(m_1, m_2\right) - t m_i.$$

Differentiating we get the necessary first-order conditions for an interior equilibrium

$$\rho\left(m_1, m_2\right) \cdot \frac{1}{2} \cdot \frac{h'\left(m_1\right)}{1 + \underline{\alpha}_1 + h\left(m_1\right)} - t = \frac{\partial S_1\left(m_1, m_2\right)}{\partial m_1} - t = 0 \tag{3}$$
$$\frac{1}{2} \cdot \frac{h'\left(m_2\right)}{1 + \underline{\alpha}_1 + h\left(m_1\right)} - t = \frac{\partial S_2\left(m_1, m_2\right)}{\partial m_2} - t = 0. \tag{4}$$

These conditions are also sufficient for an interior optimum because the left-hand side of (3) is decreasing in m_1 and the left-hand side of (4) is decreasing in m_2. Note that the left-hand side of (3) is smaller than the left-hand side of (4) when $m_1 = m_2$. In this sense, as discussed in the introduction, the disadvantaged party has stronger incentives to spend campaign funds compared to the advantaged party.

The first-order conditions (3) and (4) represent "reaction functions," a familiar tool of standard contest theory. The disadvantaged party's expenditures enter as a strategic complement[13] in the advantaged party's objective function: increased

[13] See Bulow *et al.* [8].

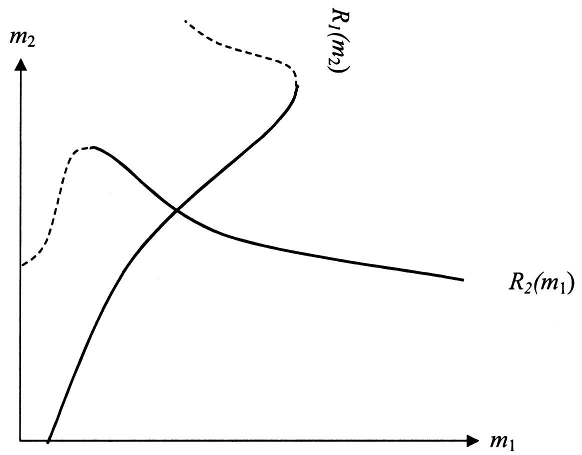

Figure 1. Reaction functions

expenditure by the underdog increases the marginal benefit of additional spending by the favored party. Conversely, the advantaged party's expenditures enter as strategic substitute in the disadvantaged party's objective function. These strategic properties will be key to our analysis of the effect of campaign spending regulation. Of immediate interest is the fact that these properties guarantee uniqueness of equilibrium in the campaign spending game.

Theorem 2 *There is a unique Nash equilibrium of the campaign spending game.*

Proof. See Appendix B. □

The intuition for the uniqueness of equilibrium can be obtained by inspecting Figure 1. That figure represents the parties' reaction functions in the space of campaign expenditures. Party i's reaction function is denoted by $R_i(m_j)$. The slope of Party 2's reaction function depends on whether Party 2 is the leader, i.e., on whether Party 2's expenditures overcompensate for its initial disadvantage. Since the identity of the leader is endogenous, it helps to partition the space into two areas, depending on which party is the leader. The upper-left region corresponds to Party 2 being the leader, the remaining portion of the space are areas where Party 1 is the leader. Correspondingly, the reaction functions are drawn with a thick line when Party 1 is the leader, and with a shaded line otherwise.

The properties of strategic complementarity imply that the leader's reaction function is increasing in the opponent's expenditure and the follower's reaction function is decreasing. An equilibrium point is a crossing of the two reaction functions. We claim that only one crossing is possible. First, the crossing of a thick and a shaded curve is inconsistent with the curves representing reaction functions, as two parties cannot simultaneously be leaders , nor simultaneously followers. Consider then crossings of thick with thick, or shaded with shaded curves. Due to the properties of strategic complementarity there can be at most one crossing of each of these two types. Furthermore, if both types of crossing coexisted, the picture shows that there would have to be a thick-with-shaded crossing, and that is not possible.

Inspection of Eqs. (3) and (4) reveals that in equilibrium the disadvantaged party outspends its opponent. Start from a point at which $m_1 = m_2$; then the left-hand side of (4) is greater than the left-hand side of (3) due to the presence of the multiplicative term ρ in the leader's first-order condition. Since $\rho \leq 1$, the marginal benefit of additional valence is greater for the disadvantaged party. As the higher benefit of valence induces the disadvantaged party to exceed its opponent's expenditure, the productivity of expenditure decreases. This effect is captured by the presence of the term $h'(m)$, which decreases in m. So, in equilibrium the disadvantaged party will outspend its opponent to the point where the higher benefit of additional valence is exactly offset by the lower productivity of spending. Incidentally, the fact that in equilibrium the initially disadvantaged party spends more shows that the initial differential in vote shares is reduced by the possibility of spending campaign funds. This suggests that campaign expenditures play a pro-competitive role in our model.

Note, as a side issue, that even if the two parties started out symmetric ($\underline{\alpha}_1 = \underline{\alpha}_2$) they would still spend funds trying to gain a valence advantage, even though in equilibrium that hope would be frustrated for both parties, similar to a Prisoner's Dilemma. If we perform the comparative statics exercise of increasing $\underline{\alpha}_2$, then Party 1's reaction function moves to the right, while party 2's reaction function remains unchanged. This means that party 2's equilibrium expenditure decreases and party 1's increases.

Finally, close examination of the first-order conditions establishes that in equilibrium Party 2 does not overtake Party 1, and therefore Eqs. (3) and (4) represent the relevant first-order conditions. Suppose Party 2 were to exceed Party 1's vote share. Then the first-order conditions (3) and (4) would hold with the indices 1 and 2 switched. In this case, at any combination $m_1 < m_2$ Party 2's marginal incentive to spend would be lower than its opponent's for two reasons: because Party 2 is now the leader (and so its incentives to spend now reflects the effect of the multiplicative term ρ) and also because $h'(m_2)$ would be smaller than $h'(m_1)$. But in equilibrium the marginal incentives to spend must be equal for both parties; therefore, equilibria in which $\rho > 1$ are ruled out.

We summarize this discussion in the following theorem.

Theorem 3 *In equilibrium Party 2 spends more than its opponent, and Party 2's vote share is larger than it would have been had both parties not spent any campaign funds. However Party* 2*'s vote share remains smaller than that of Party* 1.

In terms of Figure 1, Theorem 3 establishes that (a) the reaction functions cross above the 45 degree line; and (b) that the crossing occurs on the thick portion of the curves, as depicted.

5 Implications for campaign spending regulation

Most democracies regulate extensively campaign spending. In this section we analyze the effect of these regulations within our model. Specifically, we investigate whether regulation helps reduce the initial gap in perceived valence. We classify regulatory instruments into four main categories:

1) Floors on expenditures. These often take the form of in-kind transfers – media air-time, mailing stamps and electoral billboards – that are awarded in equal amounts to parties.

2) Spending limits, also called caps on expenditures.

3) Direct or indirect incentives to campaign spending. These exist in various forms. In many countries, part of the campaign expenditures are reimbursed either at an equal rate for all parties (matching funds) or proportionally to the vote share (or number of seats in the assembly) garnered in the election.

4) Contribution limits.

We analyze the effects of these regulatory instruments on the competitiveness of the election. We start our analysis with a benchmark case in which the function h is linear. This means that the rate at which one unit of funds is transformed into perceived valence is independent of the amount spent. In this case laissez faire is the optimal policy from the point of view of reducing the gap in perceived valence. In what follows m_1^*, m_2^* denotes the equilibrium level of spending in the absence of regulation.

5.1 A case in which laissez faire achieves perfect balance

When $h(\cdot)$ is a linear function the disadvantaged party fully catches up to the leader in the campaign spending stage and achieves a vote share of 50 percent. The reason that complete catching-up arises is equilibrium is that with linear h the productivity of spending is independent of the expenditure level. Under this assumption, therefore, the marginal incentives to spend are always greater for the disadvantaged party (the left-hand side of Eq. 4 exceeds that of Eq. 3) unless $\rho^* = 1$, which is what happens in the unique equilibrium.

To obtain a full characterization of equilibrium in the case of linear $h(\cdot)$, write $h(m) \equiv h \cdot m$. Then, the first order conditions for Parties 1 and 2 (see expression (3) and (4)) read, respectively

$$\frac{1}{2} \cdot \rho \cdot \frac{h}{1 + \underline{\alpha}_1 + hm_1} = t$$
$$\frac{1}{2} \cdot \frac{h}{1 + \underline{\alpha}_1 + hm_1} = t.$$

From the second equation we solve for Party 1's expenditure level and get

$$m_1^* = \frac{1}{2t} - \left(\frac{1 + \underline{\alpha}_1}{h} \right).$$

In order for the first equation to be verified ρ^* must equal 1. This requires choosing m_2^* so that $\underline{\alpha}_2 + hm_2^* = \underline{\alpha}_1 + hm_1^*$, or, substituting for m_1^*,

$$m_2^* = \frac{1}{2t} - \left(\frac{1 + \underline{\alpha}_2}{h} \right).$$

In this special case, a laissez-faire regulatory approach achieves $\rho^* = 1$, i.e., perfect electoral balance. Consequently, campaign finance regulation cannot help, and will generally hinder, in terms of achieving electoral balance. We now turn our attention to the case in which h is concave and so laissez faire does not achieve perfect balance.

5.2 Floors on expenditures: public funds and free advertising

We examine the practice of awarding free means of advertising, such as television time or mailing stamps, in equal amounts to both parties. Providing F dollars of free advertising alters the shape of the reaction functions. The effect can be seen in Figure 2; the reaction functions R_1 and R_2 are bounded below by F.

Imposing a floor F has no effect on the equilibrium vote shares when F is smaller than m_1^*, the unregulated equilibrium expenditure. In this case, parties will simply substitute private with public funds.

When F exceeds m_1^* Party 1 will not spend any private funds and the equilibrium lies at the intersection between R_2 and the flat portion of R_1. If in addition F is smaller than m_2^*, as depicted in Figure 2, Party 2's spending is smaller at the regulated equilibrium than at the unregulated equilibrium, and is decreasing in F. Since Party 1's expenditure equals F, Party 2's vote share decreases as F increases. In this case floors on expenditures are anti-competitive.

The effect of floors need not always be anti-competitive. As F becomes larger than m_2^* both parties forgo private expenditures and rely solely on public funds. Suppose then that $h(\infty) = \infty$, i.e., h grows without bound. Under this assumption, the vote share of the disadvantaged party approaches 50 percent as F goes to infinity. This shows that if F is sufficiently large, a floor on expenditures can be pro-competitive. However, for this to be true it is necessary that neither party finds it beneficial to spend in excess of the floor.

We summarize these results as follows.

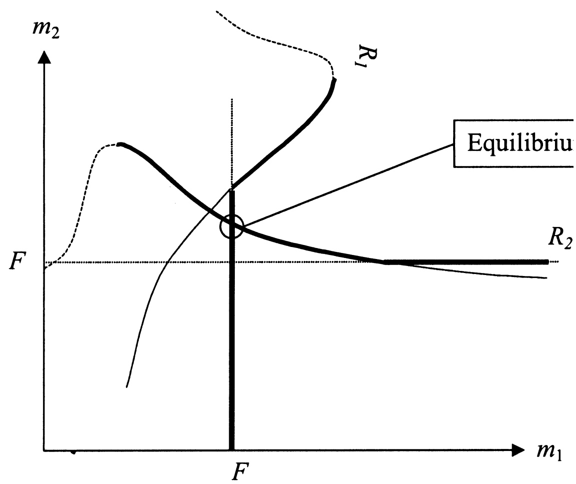

Figure 2. The effect of floors on spending

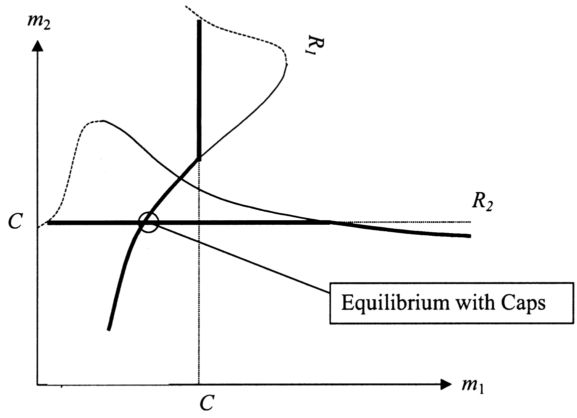

Figure 3. The effect of spending caps

Theorem 4 *Suppose the floor F is such that in equilibrium at least one party spends more than F. Then Party 2's vote share is (weakly) decreasing in F.*

5.3 Spending limits

We now examine the effect of a cap on expenditures. A cap prevents both parties from spending more than C in campaign funds. Figure 3 depicts reaction functions that are bounded above by the cap on spending.

The equilibrium with spending caps lies at the intersection of the modified reaction functions. The direct effect of spending limits is clearly anti-competitive in this model: a binding cap limits the disadvantaged party's spending and does not allow it to catch up with its opponent (R_2 is bounded above by C). There is, however, an indirect effect of imposing a cap. Since the disadvantaged party's expenditure enters as a strategic complement in Party 1's payoff, the reduction in Party 2's expenditure also leads to a decrease in Party 1's expenditure. The strategic effect on Party 1's expenditure is apparent in the picture; reducing C leads to a reduction of both m_1 and m_2 at the constrained equilibrium. In principle, therefore, the effect on vote shares could be ambiguous.

We need to establish what happens to vote shares as m_1 and m_2 move along the R_1 curve towards the origin, in response to a reduction in C. Lemma 5 shows that Party 2's vote share decreases.

Lemma 5 *Suppose Party 2 spends C and Party 1 best-responds with $R_1(C)$. Then Party 2's vote share is increasing in C.*

Proof. By assumption Party 2's vote share equals $\frac{1}{2} \cdot \rho(R_1(C), C)$. Our goal is to show that $d\rho(R_1(C), C)/dC \geq 0$. Totally differentiating ρ yields

$$\frac{d\rho\left(R_1\left(C\right), C\right)}{dC} = \frac{\partial\rho\left(R_1, C\right)}{\partial R_1} \cdot \frac{\partial R_1\left(C\right)}{\partial C} + \frac{\partial\rho\left(R_1, C\right)}{\partial C}.$$

We now explicitly compute the three derivatives that appear on the right-hand side. Differentiating expression (2) we get

$$\frac{\partial \rho (R_1, C)}{\partial R_1} = -h^{'} (R_1) \frac{1 + \underline{\alpha}_2 + h(C)}{[1 + \underline{\alpha}_1 + h(R_1)]^2},$$

and

$$\frac{\partial \rho (R_1, C)}{\partial C} = \frac{h^{'} (C)}{1 + \underline{\alpha}_1 + h(R_1)}.$$

It remains to compute $\partial R_1(C)/\partial C$. The quantity $R_1(C)$ solves the first order conditions (3)

$$h' (R_1) \frac{1 + \underline{\alpha}_2 + h(C)}{[1 + \underline{\alpha}_1 + h(R_1)]^2} - 2t = 0,$$

and the implicit function theorem yields

$$\frac{\partial R_1(C)}{\partial C} = \frac{h'(C) \cdot h'(R_1)}{[1 + \underline{\alpha}_2 + h(C)] \left(2 \frac{[h'(R_1)]^2}{1+\underline{\alpha}_1+h(R_1)} - h''(R_1) \right)}.$$

Substituting for the three derivatives and rearranging yields:

$$\begin{aligned} \frac{d\rho (R_1(C), C)}{dC} &= -\frac{h'(C) \cdot \left[h^{'}(R_1)\right]^2}{[1+\underline{\alpha}_1+h(R_1)] \left(2 [h'(R_1)]^2 - h''(R_1)(1+\underline{\alpha}_1+h(R_1)) \right)} \\ &\quad + \frac{h^{'}(C)}{1 + \underline{\alpha}_1 + h(R_1)} \\ &= \frac{h'(C)}{1 + \underline{\alpha}_1 + h(R_1)} \left[\frac{h'(R_1)^2 - h''(R_1)(1 + \underline{\alpha}_1 + h(R_1))}{2h'(R_1)^2 - h''(R_1)(1 + \underline{\alpha}_1 + h(R_1))} \right] \end{aligned}$$

Since $h'' < 0$, this expression is positive. □

Using Lemma 5 it is immediate to show that the effect of spending caps is anti-competitive. Indeed, it is graphically evident that, to the extent that caps affect the equilibrium, the equilibrium point lies either on the unmodified reaction function of Party 1 (above the 45^0 line, as depicted in Fig. 3) or on the 45^0 line.[14] In the first case, Lemma 5 applies. In the second case Party 2's vote share is $\rho(C, C)$, which is increasing in C. In both cases, therefore, spending caps have anti-competitive effects. This argument is summarized in Theorem 6.

Theorem 6 *Suppose a spending limit of C is imposed, then Party 2's vote share is (weakly) increasing in C.*

[14] This property follows from the fact that the unmodified reaction functions R_1 and R_2 cross above the 45 degrees line, together with the fact that strategic complementarities guarantee that the relevant portion of R_1 (respectively, R_2) is increasing in m_2 (resp., decreasing in m_1).

5.4 Matching funds, per-seat reimbursement, and contribution limits

In this section we analyze three common forms of public subsidy to parties: the practice of matching private with public funds, the reimbursement to parties based on their vote shares (this reimbursement often takes the form of money transfers proportional to the number of seats in the assembly), and contribution limits. We model contribution limits simply as increasing the opportunity cost of funds, so Theorem 8 below immediately applies to contribution limits.[15] We now turn our attention to matching funds and per-seat reimbursement.

The two policy instruments, matching funds and per-seat reimbursements, can be ranked within our model. The two instruments span the same policy space, in the sense that the incentive effects of matching funds can be replicated by choosing an appropriate amount of per-seat reimbursement. Furthermore, any given incentive scheme that can be implemented via a mix of matching funds and per-seat reimbursement is implemented more cheaply via the exclusive use of matching funds.

To verify that matching funds and per-seat reimbursement span the same policy space, consider a system involving the two policies; the government matches funds, i.e., reimburses s cents for each dollar spent by the party, and also reimburses r dollars per seat obtained (for simplicity, we normalize the size of the assembly to 100 seats and assume that seats are awarded in exact proportion to a party's vote share). Party i's maximization problem reads

$$\max_{m_i} \ (1+r)\, S_i\left(m_i, m_j^*\right) - (t-s)\, m_i.$$

The first-order conditions for this problem are

$$\frac{\partial S_i\left(m_i, m_j^*\right)}{\partial m_i} = \frac{(t-s)}{(1+r)}.$$

Since the first-order conditions depend on s and r only through the ratio $(t-s)/(1+r)$, the effect on Party i's behavior of setting $s>0$ can be achieved by choosing $s=0$ and a suitably larger value for r. This shows that matching funds and per-seat reimbursement span the same policy space.

To see that any given m_i^* can be implemented most cheaply only using matching funds, let $(\widetilde{r}, \widetilde{s})$ denote a particular combination of instruments that induces Party i to spend m_i^*. Substitute from the first-order conditions into Party i's maximization problem to get the value of Party i's problem,

$$(1+\widetilde{r}) \left[S_i\left(m_i^*, m_j^*\right) - \left.\frac{\partial S_i\left(m_i, m_j^*\right)}{\partial m_i}\right|_{m_i=m_i^*} m_i^* \right].$$

[15] It might be argued that contribution limits also make the cost function more convex. Since we show below that the effects of matching funds depend on the curvature of the advertising production function, adding another dimension in the convexity of the cost function would make the analysis difficult.

Because S_i is strictly concave in m_i we know that the expression in brackets is positive and therefore the value of Party i's problem is increasing in $\widetilde{r}$. In other words, among all combination $(\widetilde{r}, \widetilde{s})$ that induce Party i to spend m_i^*, the value of Party i's problem is higher when $\widetilde{r}$ is higher. Since at all such combinations Party i obtains the same vote share and spends the same amount of private funds, a higher value must reflect larger subsidies. From the viewpoint of financing the subsidy, therefore, it is cheapest to induce m_i^* by setting $\widetilde{r} = 0$.[16] We collect these arguments in the following result.

Theorem 7 *Any pair m_i^*, m_j^* that is achieved through a per-seat reimbursement can be achieved more cheaply by promising matching funds.*

The channel through which changes in s and r affect the equilibrium is through changing the marginal cost of funds for parties. For example, providing matching funds has the same effect on equilibrium vote shares as reducing t. Knowing how the equilibrium changes with t, therefore, is technically equivalent to evaluating the effect of changes to s and r. The next result deals with changes in the equilibrium due to changes in t. Although reducing t makes it cheaper for the disadvantaged party to spend, the advantaged party also spends more as a result of the subsidy. In turn, the fact that the advantaged party spends more has a negative strategic effect on the disadvantaged party's expenditure. We now show that the total effect of reducing the marginal cost of funds can be pro- or anti-competitive, depending on the curvature of the function h.

Theorem 8 *As the opportunity cost of funds t becomes smaller (due to matching funds or to a per-seat subsidy),*

(i) both parties spend more
(ii) if $h^{''}(\cdot)/h^{'}(\cdot)$ is increasing, the vote share of Party 2 increases
(iii) if $h^{''}(\cdot)/h^{'}(\cdot)$ is decreasing, the vote share of Party 2 decreases.

Proof. Denote by m_i^* the equilibrium expenditures for given values of $\underline{\alpha}_i$ and t. To prove part (i) we compute $\partial m_2^*/\partial t$ and $\partial m_1^*/\partial t$. The computation involves applying the Implicit Function Theorem to the system of first-order conditions (3) and (4), and is relegated to appendix C. There, we show that

$$\begin{bmatrix} \dfrac{\partial m_1^*}{\partial t} \\[2ex] \dfrac{\partial m_2^*}{\partial t} \end{bmatrix} = \frac{2\alpha_1^*}{D} \begin{bmatrix} h^{'}(m_1)\, h^{'}(m_2) - \alpha_1^* h^{''}(m_2) \\ h^{'}(m_1)\, h^{'}(m_2) - \alpha_2^* h^{''}(m_1) \end{bmatrix}, \tag{5}$$

where $D < 0$ and we denote $\alpha_1^* = (1 + \underline{\alpha_1} + h(m_1))$ and $\alpha_2^* = (1 + \underline{\alpha_1} + h(m_2))$. This shows that $\partial m_i^*/\partial t < 0$, which proves part (i).

[16] Although in our simple model matching funds dominate per-seat reimbursements, the latter may be desirable for reasons that are not captured by our model. One advantage of per-seat reimbursement schemes is that their cost is pre-determined independently of the parties' campaign expenditures.

To prove part (ii), observe that

$$\frac{d\rho\left(m_1^*, m_2^*\right)}{dt} = \frac{\partial\rho\left(m_1^*, m_2^*\right)}{\partial m_1^*}\frac{\partial m_1^*}{\partial t} + \frac{\partial\rho\left(m_1^*, m_2^*\right)}{\partial m_2^*}\frac{\partial m_2^*}{\partial t}$$
$$= 2t\left(\frac{\partial m_2^*}{\partial t} - \frac{\partial m_1^*}{\partial t}\right),$$

where the second equality follows from the first-order conditions of the campaign spending game. Thus, we have

$$sgn\left(\frac{d\rho\left(m_1^*, m_2^*\right)}{dt}\right) = sgn\left(\frac{\partial m_2^*}{\partial t} - \frac{\partial m_1^*}{\partial t}\right)$$
$$= sgn\left(\alpha_2^* h^{''}\left(m_1^*\right) - \alpha_1^* h^{''}\left(m_2^*\right)\right),$$

where the last equality follows from (5). From the first-order conditions (3) and (4) we have

$$\frac{\alpha_2^*}{\alpha_1^*} = \frac{h^{'}\left(m_2^*\right)}{h^{'}\left(m_1^*\right)},$$

and so

$$sgn\left(\frac{d\rho\left(m_1^*, m_2^*\right)}{dt}\right) = sgn\left(\frac{h^{''}\left(m_1^*\right)}{h^{'}\left(m_1^*\right)} - \frac{h^{''}\left(m_2^*\right)}{h^{'}\left(m_2^*\right)}\right).$$

To conclude the proof of part (ii), remember that $m_1^* \leq m_2^*$ by Theorem 3, so $d\rho^*/dt < 0$ if $h^{''}(x)/h^{'}(x)$ is increasing. The proof of part (iii) is analogous to that of part (ii). □

There is a simple intuition for the role played in Theorem 8 by the curvature of the function $h(\cdot)$. As the opportunity cost of funds decreases, equilibrium spending increases moving both parties further along the function $h(\cdot)$. Moving along the $h(\cdot)$ curve decreases the marginal productivity of spending for parties (h is concave), and hence their incentives to advertise. When $h^{''}(x)/h^{'}(x)$ is increasing, the return to expenditure decreases at a decelerating rate;[17] that is, the returns to spending for the party which spends more do not decrease as much as they do for its opponent. In these circumstances, it is not surprising that reducing the marginal cost of spending has a stronger effect on the expenditure of the party which already spends more. In our model, that party is the disadvantaged one, so reducing the marginal cost of spending has pro-competitive effects.

When the rate is accelerating, the opposite effect is at play. In practice, and since in this paper we take the function h as a primitive, it is hard to make a case for either the accelerating or the decelerating case. However, in terms of the implications for public regulation this is a potentially important feature of the productivity of campaign funds.

[17] The ratio $h^{''}/h^{'}$ is negative and achieves its maximum possible value of 0 when h is linear. When h is linear the marginal return to spending does not decrease at all.

6 Polarization

In this section we explore the redistributional and welfare implications of the difference in valence. In Section 3 we saw that the party with lower perceived valence runs on a platform that creates inequality and that a fraction of the electorate is completely expropriated. The fraction of the electorate that is completely expropriated is increasing in the difference in valence. It therefore seems plausible that the outcome would be more unequal when the two parties are more different in perceived valence. To translate this intuition into a rigorous welfare argument we need to take a stand on the welfare effects of perceived valence. In what follows, we take the position that perceived valence should be given no weight in the welfare analysis. Accordingly, in the welfare analysis we associate the same utility value to a promise of a fraction x of the budget irrespective of whether the promise comes from the party with lower or greater perceived valence. At the end of the section we show that the conclusions are preserved if we discount the utility value of promises by the party with lower perceived valence.

Definition 2 *Budget allocation F is* more fiscally polarized *than budget allocation G if G dominates F in the sense of second-order stochastic dominance.*

Under this definition, fiscal polarization is equated to inequality in redistribution. It is immediate to verify that F_2^* in Theorem 1 is more fiscally polarized than F_1^*. Moreover, denote by $F_2^*(\rho)$ and $F_2^*(\rho')$ the equilibrium platforms associated to the two values $\rho < \rho'$; then, it is easy to see that $F_2^*(\rho)$ is more polarized than $F_2^*(\rho')$. Thus, the larger the disparity in perceived valence (as measured by ρ), the larger the polarization of the disadvantaged party's budget allocation.

We now use the ranking over budget allocations to rank outcomes of the election. Our first task is to define an outcome. We build on the notion of probabilistic compromise presented in Section 2.1: with probability S_i, Party i receives power and implements its platform. For a generic voter *ex ante*, before he/she receives electoral promises from both parties, probabilistic compromise implies that the probability of receiving a transfer of less than x equals $S_1F_1(x) + S_2F_2(x)$.

Definition 3 *Given budget allocations F_1 and F_2 giving rise to vote shares S_1 and S_2, we define an* outcome *as the budget allocation $S_1F_1 + S_2F_2$.*

Our goal is to rank outcomes as ρ varies. As ρ decreases, two things happen. First, while F_1 stays constant, F_2 becomes more polarized. Second, S_2 decreases. The first effect tends to make the allocation more polarized, the second effect works in the opposite direction by reducing the relative weight of F_2. The next theorem shows that as long as the coefficient ρ is not too small (that is, the difference in perceived valence between the two parties is not too large) it is possible to rank outcomes according to polarization, and the ranking has the expected property that larger values of ρ induce outcomes that are less polarized.

Theorem 9 *Assume $1/2 \leq \rho \leq \rho' \leq 1$. Then the outcome of an election with a perceived valence ratio (PVR) of ρ is more fiscally polarized than the outcome of an election with a PVR of ρ'.*

Proof. Given any ρ denote the corresponding outcome by $F_\rho(x)$,

$$F_\rho(x) \overset{\text{def}}{=} S_1 \cdot F_1^*(x) + S_2 \cdot F_2^*(x),$$

where S_1, S_2, F_1^*, and F_2^* depend on ρ as stated in Theorem 1. There, it is shown that $F_1(x) = x/2$ for x in $[0, 2]$, and $F_2(x) = 1 - \rho + x \cdot (\rho^2/2)$ for x in $[0, 2/\rho]$. Substituting, we obtain

$$F_\rho(x) = \begin{cases} 0 & \text{for } x < 0 \\ \left(1 - \frac{\rho}{2}\right) \frac{\min\{x, 2\}}{2} + \frac{\rho}{2}\left(1 - \rho + x\frac{\rho^2}{2}\right) & \text{for } 0 \le x \le \frac{2}{\rho} \\ 1 & \text{for } \frac{2}{\rho} < x \end{cases} \cdot$$

The c.d.f. $F_\rho(x)$ has a jump at 0, is continuous on $(0, \infty)$ and linear on the two intervals $[0, 2]$ and $[2, 2/\rho]$. The function has a kink at $x = 2$, where it becomes less steep. We now show that, for $\rho' > \rho \ge 1/2$, $F_{\rho'}$ dominates F_ρ in the sense of second order stochastic dominance. To this end, it suffices to show that the two functions cross only once, and that at the crossing $F_{\rho'}$ crosses F_ρ from below.

The function $F_{\rho'}(x)$ achieves 1 at $x = 2/\rho' < 2/\rho$. Thus, $F_{\rho'}(x) \ge F_\rho(x)$ for $x \ge 2/\rho'$. The largest x at which $F_{\rho'}(x)$ and $F_\rho(x)$ cross may belong to the interval $[2, 2/\rho']$, in which case $F_{\rho'}(2) \le F_\rho(2)$, or to the interval $[0, 2]$, in which case $F_{\rho'}(2) \ge F_\rho(2)$. In both cases, however, the fact that both c.d.f.'s change slope at 2 guarantees that the crossing point is unique if $F_{\rho'}(0) < F_\rho(0)$. Thus, it suffices to show that $F_\rho(0)$ is decreasing in ρ. Since $F_\rho(0) = \rho(1-\rho)/2$, $F_\rho(0)$ is decreasing in ρ when $\rho > 1/2$. □

The previous theorem implies that voters with a concave utility function who ranked outcomes would prefer the outcome of a close electoral game to one of a lopsided one. This conclusion, of course, is dependent on the particular definition of outcome used. An alternative definition of outcome is one in which Party 2's allocations are discounted relative to Party 1's by the same amount that voters discount them. To render a voter indifferent to a promise of x from Party 1, Party 2 needs to promise x/ρ. Using Party 1's promises as numeraire, therefore, the distribution of benefits is given by $\widetilde{F}_\rho(x) = S_1 F_1(x) + S_2 F_2(x/\rho)$. If we take this expression as our definition of an outcome, we can inquire about ranking the distributions $\widetilde{F}_\rho(x)$ as ρ changes. One can directly verify that, given any $1/2 \le \rho \le \rho' \le 1$, $\widetilde{F}_{\rho'}(x)$ dominates $\widetilde{F}_\rho(x)$ in the sense of first-order stochastic dominance.[18] Thus, the notion that voters prefer closer races is maintained under this alternative definition of outcome.

[18] Substituting for S_1, S_2, F_1^*, and F_2^* from Theorem 1 one finds that $\widetilde{F}_\rho(x)$ is a function that jumps up to $[\rho(1-\rho)]/2$ at $x = 0$, and then increases linearly reaching 1 at $x = 2$. Therefore, $\widetilde{F}_{\rho'}(x)$ dominates $\widetilde{F}_\rho(x)$ in the sense of first order stochastic dominance if and only if $\rho'(1-\rho') < \rho(1-\rho)$. A sufficient condition for this inequality to hold is that $1/2 \le \rho$.

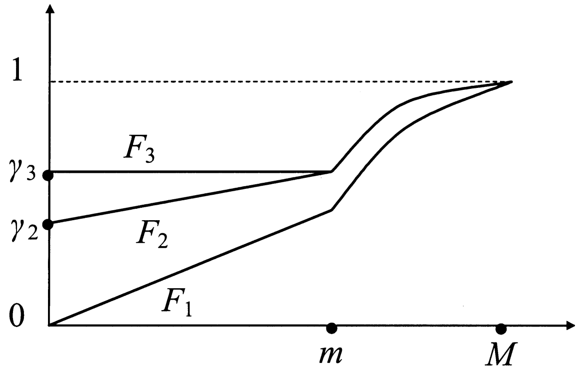

Figure 4

7 The electoral game with 3 parties

We extend the model to more than 2 parties. Let there be 3 parties with ideological performance indices (or equivalently budget constraints) α_i. Let $\alpha_1 > \alpha_2 > \alpha_3$, so that party 1 is the most favored party. Let party i distribute money to voters according to F_i. As before, each voter selects the party that promises him the largest transfer. Then, $F_{-i}(x) \equiv \underset{j \neq i}{\Pi} F_j(x)$ is the probability that party i wins a voter to whom it promised a transfer of x.

In the equilibrium of the electoral game, party i solves

$$\max_{F_i} \int_0^\infty F_{-i}(x) dF_i(x) \text{ subject to } \int_0^\infty x dF_i(x) \leq 1 + \alpha_i.$$

What will the equilibrium look like? If player i is to be willing to mix on a given support, $F_{-i}(x)$ must be linear affine on that support. With this property in mind, we conjecture that in equilibrium the F_i's must have the shape depicted in Figure 4.

To see why these strategies constitute an equilibrium, let's first focus on the interval $(0, m)$. On that interval, F_{-1} and F_{-2} are both affine linear, which implies that the parties 1 and 2 are indifferent as to how to allocate their promises on the interval $(0, m)$. In contrast, F_{-3} is convex on $(0, m)$ and so Party 3 prefers to allocate any money at 0 or m, but not in the interior of that interval.

Let us now turn to the interval $[m, M]$. On that interval, we specify $F_2(x) = F_3(x) = \sqrt{ax + b}$, which guarantees that $F_{-1}(x) = ax + b$ (an affine linear function). Moreover, we specify $F_1(x) = x/(M\sqrt{ax + b})$, which guarantees that $F_{-2}(x) = F_{-3}(x) = x/M$ (a linear function). This means that all parties are indifferent as to how to allocate any money that they distribute in the interval $[m, M]$.

The parameters a and b can be solved for using the boundary conditions $F_2(m) = \gamma_3$ and $F_2(M) = 1$. One obtains

$$a = \frac{1 - (\gamma_3)^2}{M - m} \tag{6}$$

$$b = \frac{M (\gamma_3)^2 - m}{M - m}. \tag{7}$$

The vote shares at equilibrium can be calculated noting that, wherever parties $i = 2, 3$ distribute money, they face a function $F_{-i}(x) = x/M$. Therefore, independently of how they distribute their money, their vote share is $(1 + \alpha_i)/M$. Party 1 faces an affine linear function $F_{-1}(x)$ with intercept $\gamma_2\gamma_3$. Therefore, that party receives a vote share of $\gamma_2\gamma_3 + [(1 - \gamma_2\gamma_3)(1 + \alpha_1)/M]$. Since the vote shares of parties 1,2, and 3 must add up to 1, in equilibrium it must be

$$1 = \left[\gamma_2\gamma_3 + (1 - \gamma_2\gamma_3)\left(\frac{1 + \alpha_1}{M}\right)\right] + \left[\frac{1 + \alpha_2}{M}\right] + \left[\frac{1 + \alpha_3}{M}\right]. \tag{8}$$

Finally, the equilibrium strategies must satisfy the parties' budget constraint. This means that $\int_0^M (1 - F_i(x))dx = (1 + \alpha_i)$. After solving the integrals, the budget constraint for the three parties are given by the following equations.

$$1 + \alpha_1 = M - \frac{m^2}{2M\gamma_3} - \frac{2}{3Ma^2}\left[a\left(M - m\gamma_3\right) + 2b\left(\gamma_3 - 1\right)\right], \tag{9}$$

$$1 + \alpha_2 = m\left(1 - \gamma_3\right) + \frac{m}{2}\left(\gamma_3 - \gamma_2\right) + \left[\left(M - m\right) + \frac{2}{3a}\left[\left(\gamma_3\right)^3 - 1\right]\right], \tag{10}$$

$$1 + \alpha_3 = m\left(1 - \gamma_3\right) + \left[\left(M - m\right) + \frac{2}{3a}\left[\left(\gamma_3\right)^3 - 1\right]\right]. \tag{11}$$

The six Eqs. (6) through (11) represent necessary conditions for the existence of an equilibrium of the type we have constructed.

The analysis takes as given the parameters $\alpha_1, \alpha_2, \alpha_3$ and thus allows us to draw certain parallels between the equilibrium of the two-parties and the three parties electoral stage game. First, since $S_2/S_3 = (1+\alpha_2)/(1+\alpha_3)$ and $S_1/S_i > (1 + \alpha_1)/(1 + \alpha_i)$, we conclude that party 1 enjoys an advantage in equilibrium disproportionate to its advantage in terms of popularity. Second, the introduction of a third party leads all parties to promise the most favored voters more money compared to the case of two parties (formally, M is always higher than $(1+\alpha_1)/2$). Because of this competitive effect, the introduction of a third party leads party 2 to increase the fraction of voters to which it promises nothing. Since the third party is even more likely than party 2 to promise a voter nothing, and since the introduction of a third party decreases the vote share of party 1 relative to the two-party case, introducing a third party increases the chance of a generic voter ending up with no transfers in the equilibrium. Overall, for a given configuration of parameters α_1, α_2, the effect of introducing a third party with popularity α_3 appears to have a polarizing effect on the distribution of promises.

Our ultimate goal, of course, would be to derive $\alpha_1, \alpha_2, \alpha_3$ as endogenous variables determined in the equilibrium of the campaign spending game. To achieve this goal, we need explicit solutions for the vote shares S_i as a function of the parameters $\alpha_1, \alpha_2, \alpha_3$. This seems to require solving the system of Eqs. (6) through (11) analytically, which we are currently unable to do.[19] We leave this important extension for future research. The analysis of the 3-parties electoral stage presented here provides some preliminary insights into the equilibrium of the electoral stage.

[19] We were able, however, to solve numerically for the six endogenous parameters $a, b, M, m, \gamma_2, \gamma_3$ after arbitrarily fixing the values of the three primitive parameters $\alpha_1, \alpha_2, \alpha_3$.

8 Conclusion

We have analyzed a tractable model of electoral competition in which parties compete by making redistributive promises to voters. Before the electoral competition takes place, parties have the option of spending resources to increase their perceived valence. Our model has certain features, which we view as capturing the essential elements in the debate on campaign spending: (a) one party starts out being reputed "less valent" by voters, and so is at a disadvantage; (b) campaign spending increases the voters' perception of a party's valence, and (c) equal expenditures for both parties tend to equalize vote shares; (d) parties have access to "private" funds (at some cost), which they can use to integrate any public subsidy.

Campaign spending regulation has an important role to play in this framework. Our analysis has distinguished among various familiar instruments of campaign spending regulation. Caps and floors on expenditure, and subsidies such as per-seat reimbursements can have very different consequences on the outcome of the election. Our model can provide some evaluation of possible changes in the campaign spending regulation. One important message of this paper is that, when evaluating the effect of regulations, we cannot take as given the parties' behavior. Parties will generally invest their "private" resources on top of any public monies they receive. Since the amount of private resources invested is endogenous, any change in regulation will result in adjustments by the parties. We have demonstrated the presence of a catching-up effect which causes the disadvantaged party to invest more "private" resources than the advantaged party. A laissez-faire approach has the virtue of allowing disadvantaged party to catch up, with the resulting welfare benefits deriving from a decreased polarization of platforms. In this context constraining campaign spending tends to be anti-competitive and welfare-reducing, and many campaign reforms have the effect of increasing polarization. In this dimension, our conclusions markedly differ from the view that political parties should be restrained from raising and spending "private" resources, in order to avoid socially inefficient policy distortions in favor of the lobbies who provide these resources.

We conjecture that a simple modification of the two-party analysis can shed light on the majoritarian (with two parties) case. For example, suppose that parties maximize the probability of winning but there is a probability that even the party with lower valence gets into office, perhaps due to some electoral shock. In this scenario, the parties would be maximizing a *nonlinear* function of their vote shares, instead of the linear function we assumed. As long as the nonlinearity is not too extreme, our analysis will continue to apply, with appropriate modifications.

Campaign spending is a complex phenomenon, and our conclusions rest on the modelling assumptions. In order to concentrate on the effect of campaign spending on initial difference in perceived valence, we have made the important assumption that both parties have the same opportunity cost of funds. In this we have followed the modeling assumptions of Dixit [12] and Baik and Shogren [4]. We view this assumption as a reasonable benchmark in a model of parliamentary democracy, in which "incumbency" is less of a factor than in the elections of US representatives. Even with this simplifying assumption, the model is quite rich in its consequences for campaign spending regulation. Of course, even in parliamentary democracies it is likely that parties with larger perceived valence find it easier to raise funds. Thus, caution needs to be exercised when drawing from this paper to discuss the policy implications of the effect of specific financing rules on the competitiveness of elections.

Our analysis has mostly ignored the direct social loss associated with campaign expenditures, namely, the fact that money is wasted on advertising. We think that as a first pass this is a reasonable choice given the relatively modest outlays that pertain to campaign spending relative to the implications for fiscal redistribution on which this paper focuses.[20] Of course, the question of minimizing campaign expenditure remains interesting, and the model presented in this paper could be used to address that question.

Appendix

A Connection with all-pay auctions

In Section 3 we solved for the equilibrium of a game of electoral redistribution. We found that the disadvantaged party stays out of the competition for some voters by promising of zero to a fraction of the electorate. Consider another, apparently unconnected game, a complete information all-pay auction game between two bidders one of whom is commonly known to value the object more than the other. This game is studied by Baye et al. (1993), who show that the bidder who values the object less stays out (bids zero) with some positive probability. The similarity between the all-pay auction and the game of electoral redistribution goes further: there is a one-to-one correspondence between the equilibria of the two games. We establish this correspondence by showing that the optimization problems faced by players in both games are the same up to a renormalization. This also allows us to prove the uniqueness of the equilibrium of the elction game.

Using Eq. (1), and denoting $\hat{F}_2(x) = F_2(x/\rho)$, we get Party 1's best response problem:

$$\max_{F_1} \int_0^\infty \hat{F}_2(x) dF_1(x) \text{ subject to } \int_0^\infty x dF_1(x) \leq 1.$$

The associated Lagrangian is:

$$\begin{aligned} &\max_{F_1} \int_0^\infty \hat{F}_2(x) dF_1(x) + \lambda_1^* [1 - \int_0^\infty x dF_1(x)] \\ = \lambda_1^* \; &\max_{F_1} \int_0^\infty (\frac{1}{\lambda_1^*} \cdot \hat{F}_2(x) - x) dF_1(x) + \lambda_1^*. \end{aligned} \tag{12}$$

Similarly, Party 2's best response problem is:.

$$\max_{F_2} \int_0^\infty F_1(\rho x)\, dF_2(x) \;\text{ s.t. } \int_0^\infty x dF_2(x) \leq 1.$$

[20] This point has been made, among others, by Levitt [21].

Applying the change of variable $y = \rho x$ we rewrite the above problem as

$$\max_{\hat{F}_2} \int_0^\infty F_1(y)\, d\hat{F}_2(y) \text{ s.t. } \int_0^\infty y d\hat{F}_2(y) \leq \rho.$$

The associated Lagrangian is:

$$\max_{\hat{F}_2} \int_0^\infty F_1(y)\, d\hat{F}_2(y) + \lambda_2^* \left[\rho - \int_0^\infty y d\hat{F}_2(y)\right]$$
$$= \lambda_2^* \cdot \max_{\hat{F}_2} \int_0^\infty \left[\frac{1}{\lambda_2^*} \cdot F_1(y) - y\right] d\hat{F}_2(y) + \lambda_2^* \rho. \tag{13}$$

Up to linear affine transformations involving the Lagrange multipliers, expressions (12) and (13) can be interpreted as the payoffs of bidders who bid according to mixed strategies $F_1(\cdot)$ and $\hat{F}_2(\cdot)$ in an all-pay auction for a prize that they value respectively

$$v_1 = \frac{1}{\lambda_1^*} \tag{14}$$
$$v_2 = \frac{1}{\lambda_2^*}. \tag{15}$$

Thus, every equilibrium strategy combination in the election is also the unique (see [7]) equilibrium in a certain associated all-pay auction. Each equilibrium in the election gives rise to a pair of Lagrange multipliers $(\lambda_1^*, \lambda_2^*)$ and, through (14,15), to an associated pair of valuations (v_1, v_2) giving rise to an all-pay auction. We now show that only a specific pair (v_1, v_2) gives rise to equilibrium strategies (for the all-pay auction) that meet the budget constraint of the parties in the election. This will prove uniqueness of equilibrium in the election. To this end, observe that in the equilibrium of an all-pay auction with values v_1, v_2, bidder 1 randomizes uniformly on $[0, v_2]$ (see [7]). This means that bidder 1 spends $v_2/2$. From the budget constraint in the election, we know that party 1 must spend \$1 in equilibrium, which means that in every all-pay auction corresponding to an equilibrium in the election, it must be

$$v_2 = 2. \tag{16}$$

Turning now to bidder 2, we know from [7] that bids 0 with probability $(v_1 - v_2)/v_1$ and with complementary probability randomizes uniformly on $[0, v_2]$. This means that bidder 1 spends $v_2^2/2v_1$. From the budget constraint in the election, we know that party 2 must spend \$$\rho$ in equilibrium, which means that in every all-pay auction corresponding to an equilibrium in the election, it must be $v_2^2/2v_1 = \rho$. Substituting from Eq. (16) and solving for v_1 yields

$$v_1 = \frac{2}{\rho}. \tag{17}$$

Expressions (16) and (17) identify a unique all pay auction with a unique equilibrium. Any equilibrium in the election must be an equilibrium in this all pay auction. Thus, the equilibrium in the election is unique.

B Proof of Theorem 2

Existence follows from standard arguments. We now show that the equilibrium of the spending game is unique. Our assumption that $h'(0) = \infty$ implies that any equilbrium is interior. Suppose there are two interior equilibria, (m_1, m_2) and (m'_1, m'_2), and let us show that it must be $\rho(m_1, m_2) = \rho(m'_1, m'_2)$. Suppose not, and without loss of generality let $\rho(m_1, m_2) < \rho(m'_1, m'_2)$. Then, in view of the first order conditions for party 1 (see expression (3)) we have

$$\frac{1}{2}\rho\left(m_1, m_2\right)\frac{h'\left(m_1\right)}{1+\underline{\alpha}_1+h\left(m_1\right)} = t = \frac{1}{2}\rho\left(m'_1, m'_2\right)\frac{h'\left(m'_1\right)}{1+\underline{\alpha}_1+h\left(m'_1\right)}. \tag{18}$$

Since by assumption $\rho(m_1, m_2) < \rho(m'_1, m'_2)$, it must be that $\frac{h'(m_1)}{1+\underline{\alpha}_1+h(m_1)} > \frac{h'(m'_1)}{1+\underline{\alpha}_1+h(m'_1)}$, and thus $m'_1 > m_1$. From the first order conditions for party 2 (see expression (4)) we have

$$\frac{1}{2}\frac{h'\left(m_2\right)}{1+\underline{\alpha}_1+h\left(m_1\right)} = t = \frac{1}{2}\frac{h'\left(m'_2\right)}{1+\underline{\alpha}_1+h\left(m'_1\right)}, \tag{19}$$

whence, since $m'_1 > m_1$, we get $m'_2 < m_2$. However, this configuration of investments is inconsistent with the assumption that $\rho(m_1, m_2) < \rho(m'_1, m'_2)$. Thus, given two equilibria, there cannot be one that gives rise to a larger ρ than the other.

Having shown that all equilibria give rise to the same value of ρ, notice that given a value ρ^*, the first order conditions uniquely pin down the equilibrium m_1^*. Then, m_2^* is determined by the equation $\rho(m_1^*, m_2) = \rho^*$. This concludes the proof that there cannot be two interior equilibria.

C Derivation of $\partial\mathbf{m}^*/\partial t$

The first-order conditions of the parties can be expressed as

$$\mathbf{G}\left(\mathbf{m}, t\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where

$$\begin{aligned}\mathbf{G}\left(\mathbf{m}, t\right) &= \begin{bmatrix} G_1\left(m_1, m_2, t\right) \\ G_2\left(m_1, m_2, t\right) \end{bmatrix} \\ &\stackrel{\text{def}}{=} \begin{bmatrix} h'\left(m_2\right) - 2t\left(1+\underline{\alpha}_1+h\left(m_1\right)\right) \\ \left(1+\underline{\alpha}_2+h\left(m_2\right)\right)h'\left(m_1\right) - 2t\left(1+\underline{\alpha}_1+h\left(m_1\right)\right)^2 \end{bmatrix}.\end{aligned}$$

Apply the implicit function theorem to get

$$\frac{\partial\mathbf{m}^*}{\partial t} = -\left[\frac{\partial\mathbf{G}\left(\mathbf{m}, t\right)}{\partial\mathbf{m}}\right]^{-1}\frac{\partial\mathbf{G}\left(\mathbf{m}, t\right)}{\partial t}. \tag{20}$$

After denoting $\alpha_1^* = (1 + \underline{\alpha}_1 + h(m_1))$ and $\alpha_2^* = (1 + \underline{\alpha}_2 + h(m_2))$, we have

$$\frac{\partial \mathbf{G}(\mathbf{m}, t)}{\partial t} = \begin{bmatrix} -2\alpha_1^* \\ -2\alpha_1^{*2} \end{bmatrix},$$

and

$$\frac{\partial \mathbf{G}(\mathbf{m}, t)}{\partial \mathbf{m}} = \begin{bmatrix} -2th^{'}(m_1) & h^{''}(m_2) \\ \alpha_2^* h^{''}(m_1) - 4th^{'}(m_1)\,\alpha_1^* & h^{'}(m_1)\,h^{'}(m_2) \end{bmatrix}.$$

Hence

$$\left[\frac{\partial \mathbf{G}(\mathbf{m}, t)}{\partial \mathbf{m}}\right]^{-1} = \frac{1}{\left|\frac{\partial \mathbf{G}(\mathbf{m},t)}{\partial \mathbf{m}}\right|} \begin{bmatrix} h^{'}(m_1)\,h^{'}(m_2) & -h^{''}(m_2) \\ 4th^{'}(m_1)\,\alpha_1^* - \alpha_2^* h^{''}(m_1) & -2th^{'}(m_1) \end{bmatrix}$$

with

$$\left|\frac{\partial \mathbf{G}(\mathbf{m}, t)}{\partial \mathbf{m}}\right| = -\,2th^{'}(m_1)^2\,h^{'}(m_2) - h^{''}(m_2)\,h^{''}(m_1)\,\alpha_2^* + 4th^{''}(m_2)\,h^{'}(m_1)\,\alpha_1^* < 0.$$

Substituting into (20) we get

$$\begin{bmatrix} \dfrac{\partial m_1^*}{\partial t} \\ \dfrac{\partial m_2^*}{\partial t} \end{bmatrix} = \frac{2\alpha_1^*}{\left|\frac{\partial \mathbf{G}(\mathbf{m},t)}{\partial \mathbf{m}}\right|} \begin{bmatrix} h^{'}(m_1)\,h^{'}(m_2) - \alpha_1^* h^{''}(m_2) \\ h^{'}(m_1)\,h^{'}(m_2) - \alpha_2^* h^{''}(m_1) \end{bmatrix}.$$

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