A Drawback of Electoral Competition*

Alessandro Lizzeri

New York University and CEPR†

Nicola Persico

University of Pennsylvania‡

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Abstract

In most major democracies there are very few parties compared to the number of possible policy positions held by voters. We provide an efficiency rationale for why it might be appropriate to limit the proliferation of parties. In our model, the larger is the number of parties, the greater the inefficiency

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†Department of Economics, 269 Mercer Street, New York, NY 10003. Web: http://www.econ.nyu.edu/user/lizzeria

‡Department of Economics, 3718 Locust Walk, Philadelphia PA 19104-6297. Web: http://www.ssc.upenn.edu/~persico/.

§E-mail addresses: Lizzeri <alessandro.lizzeri@nyu.edu>, Persico <persico@ssc.upenn.edu>.

of the outcome of electoral competition. The reason is that when the number of parties increases, electoral incentives push each party to focus its electoral promises on a narrower constituency, and special interest policies replace more efficient policies which have diffuse benefits.

The analysis provides a possible explanation for the existence of institutional features that limit the extent of electoral competition: thresholds of exclusion, run-off electoral systems, and majoritarian two-party political systems.

JEL classification: D82, L15

1 Introduction

Political parties seem to be few, especially when compared to the number of distinct preference profiles held by voters. The median effective number of parliamentary parties among Lijphart's 36 democracies is smaller than 3 (Lijphart 1984), and many successful democracies make do with just two main parties. Having few parties has drawbacks because it reduces the degree to which the political system can represent the heterogeneous preferences of the electorate (Lipset and Rokkan 1967) and may also reduce the pool of available political talent (Becker 1958). Why then are there so few parties in many democracies?

The proximate cause, it has been argued, is that many electoral systems are set up to counteract the tendency of parties to multiply. A national-level two-party system, for example, would arguably be hard to maintain without the single-member districts that are employed, among others, in the US and the UK.[1] Similarly, many parliamentary democracies have thresholds of exclusion that deny representation to parties with a vote share below the threshold (e.g., Germany has a threshold of 5 %). Under a threshold of exclusion, the number of parties is reduced because parties that anticipate a small vote share do not field candidates. The general point is that several frequently observed features of electoral systems

[1] For the effect of district magnitude on the number of parties see, e.g., Ordeshook and Shvetsova (1994) and Cox (1997).

act to discourage the proliferation of parties.

If true, this explanation begs the question of whether there is an efficiency rationale for controlling party proliferation.[2] One could conjecture that having a large number of parties creates government instability, which is thought to harm economic growth.[3] But to the extent that instability causes bad economic policies, the solution is not necessarily to reduce the number of parties. Rather, one ought to consider procedures of government formation and termination that favor stability. In recent empirical work, Diermeier, Eraslan, and Merlo (2001a,b) show that procedures like the constructive vote of no-confidence, a fixed interelection period, and others can help reduce government turnover for a given number of parties. If we accept that government instability can be ameliorated by changes in the procedures of government formation, then the question remains open of why some electoral systems discourage large numbers of parties.

We propose a possible answer to this question by arguing that competition among a large number of parties has a drawback. When there are many competing parties, the electoral base of each party tends to be smaller. To cater to their narrow support base, politicians find it expedient to promise pork-barrel policies

[2] It is of course possible that some of the features of electoral systems that restrict competition were put in place not for efficiency reasons, but to protect the incumbents. In this paper we focus on the possibility of an efficiency rationale.

[3] See Alesina et al. (1996) and Campos and Nugent (2002).

with narrow appeal rather than policies which benefit the electorate at large. The resulting policies benefit the supporters of the winning politician, but do not necessarily maximize aggregate welfare. For instance, politicians must choose between supporting free trade, which is efficient but whose benefits are not easily targetable by politicians, or tariffs which politicians pick to favor particular interest groups. When politicians are catering to a narrow support base, then we should expect excessive trade restrictions. Similarly, consider politicians who must choose between creating a large bureaucracy, which entails an aggregate deadweight loss but generates some benefits (jobs, etc.) that can be targeted to supporters, or a small bureaucracy, which is efficient but does not allow the politician to target largesse to supporters. Then we should expect too large a bureaucracy.[4] The idea is that projects with diffuse benefits (free trade, absence of bureaucracy) are less appealing to office-motivated politicians because their benefits are less targetable, and therefore may be underprovided by the political system. We provide a model that suggests that this distortion becomes worse as the number of competing parties increases. The reason is that, as the support base of each party becomes a smaller fraction of the electorate, so the gain from targeting a smaller subset of the electorate increases, and the temptation for politicians to engage in special-interest

[4]Bates (1981), in his analysis of policies designed to boost agricultural production in Africa, provides evidence of this phenomenon.

politics becomes greater.[5]

Intuition for the main result Suppose parties compete for office by making a binding electoral promise to each voter. Candidates can promise one of two things: to redistribute the tax revenue (\$1 per capita) across voters, or to invest all the money in a public good, in which case all voters get utility G in money units. (The term "public good" should be interpreted broadly to indicate non-targetable policies.) Candidates maximize their expected vote share. Voters are homogeneous; each voter will vote for the party promising him the most utility. If $G > 1$, surplus maximization requires that the public good be provided. But that may not be an equilibrium. Indeed, suppose that all candidates promise the public good. Then, they receive an equal share of $1/N$ of the vote. If $G < N$, a party can deviate by offering transfers of more than G to more than $1/N$ of the voters; this is feasible and is a superior strategy. Thus, it is not an equilibrium for all parties to offer the public good, and hence the equilibrium is not surplus-maximizing for $1 < G < N$. The key point is that the range of values of G that give rise to the underprovision of the public good grows with the number of parties N.

[5] This intuition is related to Myerson (1993) who, in a model of pure redistribution, shows that the redistibution becomes more unequal as the number of candidates increases. We will discuss Myerson's paper in Section 1.1.

This simple intuition is quite robust to alternative modeling choices,[6] and says that the range of environments where the public good is underprovided grows with the number of parties. For the moment, though, it is not obvious how for any *given* environment (that is, for fixed value of G) the extent of the underprovision varies with the number of parties. In the main body of the paper we show that for any given value of G, the probability that the public good is provided is decreasing in the number of parties. In other words, fix any environment, then there is worse underprovision of the public good the more intense is electoral competition. As the number of parties goes to infinity, the probability that the public good is provided converges to zero.

This intuition suggests that, within our framework, a large number of competing parties tends to result in particularistic platforms and worse underprovision of public goods. In this sense, our model can provide a rationale for the observed propensity of electoral systems to restrain the proliferation of parties. Note that the objective of this paper is not to show that electoral competition among three

[6] For instance, the argument does not depend on whether parties maximize the vote share, as in a proportional system, or the probability of winning the election, like in a majoritarian winner-take-all system, or if they are seeking vote share to improve their bargaining position in a subsequent (unmodeled) government formation stage. Also, if offering the public good allowed a party to retain some small amount of money to redistribute, the range of inefficient G's would be approximately the same.

or more parties is *necessarily* bad: we stress that a many-party system can be appropriate—in those countries, for example, with numerous ethnic and ideological cleavages that must find an expression in the political system.[7] Our contribution is to highlight the *possibility* that electoral competition can have drawbacks quite separately from instability. This is important because, to our knowledge, this point has not been made before in a formal model.

Equipped with a formal model of the drawbacks of party proliferation, we analyze some constitutional features that appear to limit the number of parties. We show that institutions such as minimal vote share thresholds, the run-off electoral system, and majoritarian systems have desirable features because they act, in different ways, to limit the extent of electoral competition. Thus, our analysis provides a potential efficiency rationale for several common electoral institutions that appear to restrict the scope of electoral competition.

The message that electoral competition can be harmful is likely to be controversial. On the other hand, there is a large literature in public economics drawing attention to the inefficiency of decision-making in legislatures.[8] The common theme in that literature is that an inefficiency arises when legislators can purchase

[7] Also, it is possible that a many-party system might help control corruption. We will return to this issue in the Conclusions.

[8] See e.g. Weingast *et al.* (1981), Chari *et al.* (1997), and Persson *et al.* (2000). For a comprehensive survey of the literature see Persson and Tabellini (2000b).

goodies for their own district and make other districts pay for them. This type of "common pool" inefficiency tends to become worse when the number of legislators increases. How is our work related to that literature? Note that the "common pool" distortion originates from a mismatch between the locally-based incentives of politicians and their power to set policy at the national level. This mismatch is absent in our model, where politicians run for national office and so when they transfer resources to one district, they internalize the fact that the other districts are made worse off. One might surmise that politicians running for national office should not suffer from the "common pool" inefficieny and should have no incentives to inefficiently target benefits.[9] Our paper shows that this optimism is not necessarily justified. In our framework, we show that even nation-wide parties have an incentive to create endogenous constituencies. These constituencies are not based on geographic boundaries but on tactical incentives; their extension changes endogenously with the number of parties. Our analysis demonstrates that the distortions of pork-barrel politics need not be solved by nationwide competition.

[9] This intuition can be verified within a probabilistic voting model (see e.g., Coughlin 1992, and Usher 1994 for a critique of probabilistic voting). We have investigated this issue in the Lindbeck-Weibull (1987) framework of (nationwide) electoral competition. Due to the properties of the voting equilibrium in the Lindbeck-Weibull framework, when voters are homogeneous no fragmentation of policy would arise as the number of candidates increases.

1.1 Related Literature

Number of parties

Myerson (1993) compares electoral systems in terms of the inequality of redistribution in a model with N parties. Myerson shows that as N increases, equilibrium redistribution becomes more unequal due to the party's incentive to target smaller subsets of the electorate. Our paper borrows the model of redistribution from Myerson (1993), and adds a public good. The presence of the public good introduces the trade-off between efficiency and targetability which is the focus of our analysis and is absent in Myerson (1993). While the nature of the distortion is different in our model, the force that drives it is similar to the one identified by Myerson.

The effect of the number of parties on political competition has been explored in the setting of spatial competition (see, for instance, Palfrey (1984), Cox (1987) , and Austen-Smith and Banks (1988)). See Shepsle (1991) for a review of this strand of the literature. In Austen-Smith and Banks (1988) the equilibrium outcome with three parties is less efficient than the one with two parties. The reason is the following. When there are three parties, the equilibrium outcome involves two large parties located symmetrically around the median and one small party located at the median. The governing coalition comprises the small party and one of the large parties, with each large party being chosen with probability 1/2. As a result, from an ex-ante perspective, the policy outcome is a lottery between the two positions that would be chosen by the alternative coalitions. All risk averse voters prefer the median policy for sure (which is the equilibrium with two parties) to such a lottery. A similar effect is present in Cox (1987). In this framework, the inefficiency comes from the fact that equilibria with more than two parties do not necessarily involve policy convergence, and policy divergence is worse for voters from an ex-ante perspective. Note that this effect is not monotonic in the number of parties. For instance, in Cox (1987), equilibria with 4 parties are ex-ante inferior to equilibria with two parties but equilibria with 6 parties can be ex-ante superior to equilibria with 4 parties.[10]

Inefficiency in elections

In our model, the inefficiency takes the form of under-provision of the public good. This is in contrast with the conventional analysis of the inefficiency of provision of public goods in democracies (see for instance Stiglitz (1988)) which displays over- or under-provision, depending on the difference between the policy preferred by the median voter and the Samuelsonian optimum. This median voter model relies on exogenous restrictions on the dimension of the policy space, such as linear taxes, and therefore (1) cannot capture the incentives toward tactical redistribution which underlie our results, and (2) does not yield clear implications

[10] Cox only characterizes equilibria with an even number of candidates. When there is an odd number of candidates there is no pure strategy equilibrium.

about the effect of the number of parties on the efficiency of the outcome.

The idea that distortions can arises when redistributional policies targeted to particular subsets of the electorate are overprovided at the expense of projects with diffuse benefits can be traced back at least to Buchanan and Tullock (1962). Lizzeri and Persico (2001) compare winner-take-all with proportional systems, and consider the effects of the magnitude of districts, in terms of public good provision. That paper only considers two-party elections. The main point of that paper is that even with two parties these systems lead to different outcomes. In contrast, here we deal with the issue of the role of electoral competition, which cannot be done in a two-parties model.

Persson and Tabellini (1999, 2000a,b) provide empirical evidence on the provision of public goods in different political systems. Milesi-Ferretti, Perotti, and Rostagno (2000) provide a different model comparing proportional and majoritarian systems, and provide empirical evidence on the effects of the degree of proportionality of electoral systems on the composition and size of public spending. They also provide an empirical finding suggesting that the number of parties has a positive effect on the fraction of public spending that corresponds to transfers. This feature is consistent with the results of this paper.

Cox (2001) discerns two reasons why a ruling group may wish to distort its electoral platform towards targetable policies. First, to appropriate surplus for itself; second, to buy electoral support from pivotal groups. He argues that the first effect is stronger when ruling groups are more securely in power, while the second effect is stronger when subgroups of the population differ in their probability of being pivotal.

Coate and Morris (1995) focus on voters' imperfect information on the effects of government policy. Besley and Coate (1998) discuss the efficiency of representative democracy where parties are citizens who, if elected, implement their favorite policies. Theirs is a model of repeated elections, and the inefficiency depends on the dynamic nature of the model. Feddersen and Pesendorfer (1996, 1997) focus on the efficiency with which large elections aggregate information that is dispersed across voters. Their finding is that elections are relatively efficient in this dimension. Finally, Besley and Coate (1997) and Osborne and Slivinski (1996) provide a different model of multi-party elections with endogenous entry by citizen-parties who have policy preferences and do not commit to electoral platforms.

Inefficiency in legislatures

Baron (1991) models the legislative process via a sequential bargaining model. In each period, a random legislator is selected to make an offer to others, which they can accept or reject. In this game the proposing legislator has an incentive to rely on pork-barrel instead of projects with diffuse benefits because all the pork-barrel that is not needed to buy off a minimal winning coalition can be appropriated by the proposer. The fact that politicians target subset of the electorate is similar to our paper. In Baron's setup, however, the inefficiency stems from the legislator's desire to appropriate goodies for his/her own district, unlike our setup where parties have no district. More importantly, the structure of Baron's game is designed to portray a specific legislative process. It does not describe parties competing in large elections, and questions concerning alternative electoral systems cannot be studied in this context.

2 Model

In this section we present the model. Section 6 provides a discussion of some of the assumptions.

2.1 Economy and Agents

There are N parties. There is a continuum of voters with measure one. The set of voters is denoted by V. There are two goods, money and a public good. The public good can only be produced by using all the money in the economy.[11]

Each voter has an endowment of one unit of money. The public good yields a utility of G to each voter. Voters have no a priori preference for either party, and

[11] This assumption can be relaxed and is discussed in Section 6.1.

have linear utility over goods.[12]

Parties make binding promises to each voter concerning the policy if elected. A party can offer to provide the public good (to all voters); alternatively, he can offer different taxes and transfers to different voters. Each voter votes for the party who promises her the greatest utility. Parties maximize their expected vote share. This assumption on the parties' objective can be interpreted as describing a *proportional system* in which the spoils of office (seats in an assembly) are divided proportionally to the share of the vote.

2.2 Game

A pure strategy for a party specifies whether it chooses to offer the public good or transfers (it cannot offer both). In the event it chooses transfers, a pure strategy specifies a promise of a transfer to each voter. Formally, a pure strategy is a function $\Phi : V \to [0, +\infty)$, where $\Phi(v)$ represents the consumption promised to voter v. The function Φ satisfies one of two conditions: either $\Phi(v) = G$ for all v's, signifying that the party offers the public good; or, $\int_V \Phi(v)dv = 1$, which is the balanced budget condition when a party offers transfers.

[12]The assumption of linear utility is made solely to simplify the notation and presentation of the results; all results hold for any increasing utility function. Section 4 extends the analysis to the case in which the utility function is concave, i.e., voters are risk averse.

There are three stages of the game:

Stage 1 Parties choose offers to voters simultaneously and independently.

Stage 2 Each voter v gets offers $(\Phi_1(v), \ldots, \Phi_N(v))$ from parties. After observing the offers, voter v votes for party i if, for all $j \neq i$, $\Phi_i(v) > \Phi_j(v)$. Ties are resolved by randomizing with equal probability.

Stage 3 Each party is allocated seats in parliament in proportion to its vote share. A parliamentary representative at random is chosen as the formateur. The implemented policy is that of the formateur's party.

In the game specified above, the decision makers are parties and they are assumed to maximize expected vote share. In what follows we will solve for the equilibrium of this game. The game could, also, be interpreted as the reduced form of a more complicated, and perhaps more realistic model in which the decision makers are party officials. In this richer model, each party is composed of many party officials. These officials obtain a (for now exogenously specified) benefit from sitting in parliament. Before the election, officials of party i bargain over a policy platform to which the party is then committed. After the election, each party is allocated seats in proportion to its vote share, and fills its share of the seats with party officials drawn at random from its ranks. In this model, it is rational for a party's officials to agree to an electoral platform that maximizes the party's vote share.[13]

A useful feature of the game we analyze is that the probability that a party's policy is implemented is equal to that party's share of the vote. This feature simplifies our analysis in two ways. First, it makes it a (weakly) dominant strategy for a voter to vote for the party who proposes the policy that is best for her.[14] Second, it simplifies the computation of the probability of provision of the public good since, under this assumption, this probability is equal to the probability that a party offers the public good.

Since we assume that the implemented policy is that of the formateur's party,

[13] This model could be further refined. The payoff from sitting in parliament can be a result of a more primitive payoff obtained from a model of government formation. We could assume that representatives, in addition to some base utility (possibly zero) from sitting in parliament, derive additional utility if they participate in government. Government is a set of (at least) 50 percent of the representatives chosen individually by the formateur. The formateur is assumed to be able to extract the gains from being in government from the individual representatives, so that in equilibrium the formateur appropriates the rents. In this environment, the expected value for a representative of being in parliament consists of the base utility plus the expected gain that accrues to that representative when he/she is chosen as the formateur. Again, it is rational for party officials to commit their party to the policy that maximizes expected vote share because it maximizes their probability of being the formateur.

[14] More precisely, this holds in any finite approximation of our model. This feature eliminates coordination equilibria in which, to avoid wasting their vote, voters ignore some candidates who in equilibrium get no votes.

a potentially important element that is missing from our model is a realistic description of the interaction between post-election coalition formation and policy implementation. In reality, it is possible that there may be interesting interactions between the electoral and legislative stage (as modeled for instance by Austin Smith and Banks 1988 in the case of a one-dimensional policy space).

3 Equilibrium

For values of G smaller than N there is no equilibrium in pure strategies. The reason can be seen most simply in the case of two parties, i.e., $N = 2$. Suppose that $G < 2$ and party 1 promises each voter the public good for sure; then party 2 can offer transfers of more than G to more than 50 % of the voters and obtain more than 50 % of the votes. This is impossible in equilibrium since each party can guarantee itself 50 % of the votes by mimicking the strategy of the other party. Suppose instead that both parties choose to offer the public good with probability zero. Consider party 1's pure strategy distribution of money. Party 2 could offer ε more to almost all voters, and finance these offers by completely expropriating a vanishing fraction of voters. This strategy ensures that party 2 obtains a vote share arbitrarily close to one hundred %.[15]

We construct and characterize a mixed strategies equilibrium. The main intu-

[15] This argument can be modified to cover the case of N candidates.

ition of the paper (page 6) does not rely on mixed strategies. We discuss this issue in more detail in Section 6.2.

A mixed strategy in this game could be a very complicated object, since the space of pure strategies is large. We discuss the case where $\Phi_i(v)$, the offer made by party i to voter v, is a realization from a random variable with c.d.f. $F_i^v : \Re_+ \to [0, 1]$.[16] Note that, even when parties use mixed strategies, each voter observes her realized promises before voting, not random variables.

We concentrate on equilibria with *equal ex ante treatment*, those in which parties treat all voters identically ex ante. That is, we analyze equilibria in "simple" strategies of the following form: party i chooses to promise the public good with probability β_i and promise money with probability $1 - \beta_i$. When parties redistribute, they draw promises to all voters from the same F_i (notice the absence of the superscript v).[17] Also, we will restrict attention to *symmetric* equilibria, equilibria is which all parties adopt the same β and F (notice the absence of the subscript i).

Due to the large number of voters, F_i is the empirical distribution of transfers by party i to voters; that is, $F_i(x)$ is the fraction of voters who receive promises below x from party i. Let us consider some examples. An F_i that is degenerate

[16] Like Myerson (1993), we too rule out correlation between offers to different voters.

[17] Of course, the fact that offers are realizations of the same random variable does not mean that each voter gets the same offer.

at 1 implies the same offer to all voters. If $F_i(x) = 0$ for $x < 0, F_i(x) = \frac{1}{2}$ for $x < 2$, and $F_i(x) = 1$ for $x \geq 2$, candidate i offers 0 to half the voters, and 2 to the remaining half. By manipulating F_i, party i is able to target transfers to fractions of the electorate.

We want to stress that there is a natural interpretation for these mixed strategies: choosing F_i should be thought of as choosing the Lorenz curve, i.e. the empirical distribution of transfers, in the population.

3.1 Fundamental Property: Linearity of Equilibrium Payoffs

In an equilibrium with equal ex-ante treatment, all voters look identical to a party in the sense that they receive electoral promises drawn from the same distribution. Then, all the information that is relevant to a party can be summarized by a function $H^*(x)$ which denotes the equilibrium probability of winning a generic voter with an offer of x. For example, $H^*(2)$ represents the probability that party i wins a generic voter if it offers that voter a consumption level of 2. To understand the relationship between the function H^* and equilibrium behavior of parties, consider the case in which parties only promise transfers, not the public good. Let F^* denote the equilibrium distribution of transfers. Then we have $H^*(x) = [F^*(x)]^{N-1}$. The latter is simply the probability that a generic voter receives offers of less than x by all $N-1$ of party i's opponents.

We now show that, regardless of the number of parties, $H^*(x)$ must be piece-wise linear. This fundamental property will later be exploited to obtain a complete characterization of the equilibrium strategies, i.e., of the probability of public good provision and of the distribution of transfers.

There is a simple intuition behind the linearity of $H^*(x)$. Since parties who choose redistribution must be willing to play a mixed strategy in equilibrium, each party must receive the same expected payoff from promising any x in the support of the mixed strategy. The cost of promising x to a voter is the opportunity cost of x units of money in terms of vote share; this opportunity cost is x/N because in a symmetric equilibrium $1/N$ is the expected vote share from spending \$1 per capita. Whenever x is an amount of money that is promised by parties in equilibrium, then the opportunity cost of promising x must equal the expected benefit. Thus, it must be $x/N = H^*(x)$, and we have argued that the following is true.

Lemma 1 *In any symmetric equilibrium with equal ex ante treatment we have* $H^*(x) = x/N$ *whenever* x *belongs to the support of* F^* *(and* $x \neq G$*), and* $H^*(x) \leq x/N$ *otherwise.*

The previous Lemma states that if in equilibrium a party promises \$2, for example, to a voter, then it expects to win that voter with probability $2/N$. If no party promises \$2 to any voter, we can infer that the probability of winning a voter with an offer of \$2 must be smaller than (or equal to) $2/N$.

3.2 When the Public Good is Inefficient: Pure Redistribution

When $G < 1$ promising the public good is a dominated strategy. Thus, the game is one of pure redistribution. To illustrate the equilibrium logic in a simple case, in this section we compute the equilibrium strategy $F^*(x)$ in the game of pure redistribution.

Since we know that $H^*(x) = [F^*(x)]^{N-1}$, and Lemma 1 suggests that $H^*(x) = x/N$, it seems straightforward to conclude that $F^*(x) = [x/N]^{1/(N-1)}$. In order to draw this conclusion, however, Lemma 1 requires us to verify that the support of $F^*(x)$ is the entire interval $[0, N]$.

To check that there are no "holes" in the support of F^*, consider that if there were holes then there would be values of x which are not promised by any party. Consider the supremum of all such values of x: in equilibrium there would have to be a party which promises to some voters the supremum, or a sum arbitrarily close to the supremum. But this would be irrational, since that party could obtain the same probability of winning those voters by promising ε less than the supremum, and would save money.

Having checked that the support of F^* has no holes, we can state our first result, which is adapted from Myerson (1993). Myerson provides an analysis of the game where there is no public good. [18]

Theorem 1 *(Myerson 1993) If* $G < 1$ *there is a unique symmetric equilibrium with equal ex ante treatment. The public good is not provided, and parties choose transfers according to the distribution* $F_N^*(x) = \left(\frac{x}{N}\right)^{\frac{1}{N-1}}$ *for* x *in* $[0, N]$*.*

3.3 When the Public Good is Efficient: Redistribution and Public Good

We now consider the case where $G > 1$. In this case the public good must be provided with positive probability in equilibrium. Indeed, suppose to the contrary that parties $2, ..., N$ never promise the public good, and therefore behave according to the equilibrium described in Theorem 1. Then, it pays for party 1 to deviate and offer the public good, since party 1 receives a share of the vote of $H^*(G) = G/N > 1/N$. Thus, in equilibrium the public good must be promised with positive probability. Furthermore, if $G < N$ there is no equilibrium where the public good is provided with probability one; if all parties were to offer the public good, offering transfers above G to more than one N-th of the voters is feasible and would be a profitable deviation. Thus, in the region where $1 < G < N$, parties randomize

[18] Myerson proves a more general result that treats any rank-scoring rule.

between promising transfers and offering the public good.

When the public good is promised with positive probability in equilibrium, no party will promise any voter transfers of value smaller than, but very close to G. Indeed, when ε is small promising a voter $G-\varepsilon$ is less effective than promising $G+\varepsilon$: the latter only costs 2ε more and delivers a discrete increment in its probability of winning the voter. This observation implies that, when the public good is offered with positive probability in equilibrium, the support of F^* could not be the entire interval $[0, N]$; there must be a "hole" in the support of F^*. Figure 1 depicts this case; no party promises transfers between k and G, consequently, the support of H^* also has a hole between k and G. The same logic as in the previous section guarantees that there are no other holes in the support of H^*.

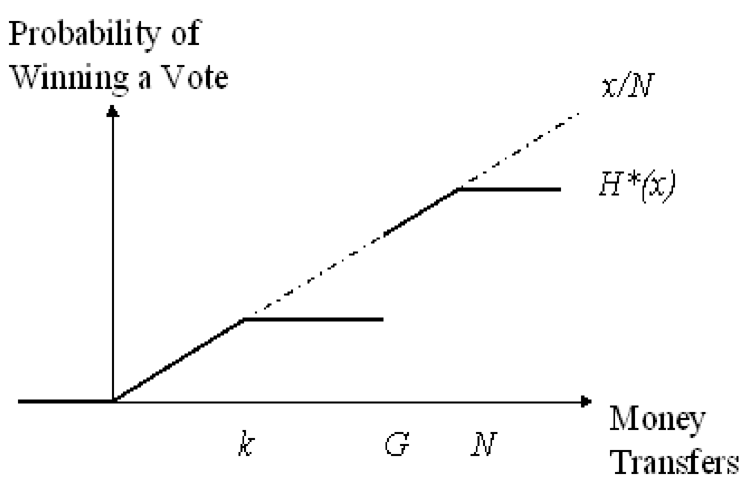

Figure 1: Returns to transfers are linear in equilibrium.

We now state the main result of the paper.

Theorem 2 ***(Existence and characterization of the equilibrium).*** *There is a unique symmetric equilibrium with equal ex-ante treatment. For $G > N$ the public good is provided with probability one. For $1 < G < N$, the equilibrium is characterized by a probability of providing the public good $\beta^* (G, N)$ which:*

(i) is increasing in G,

(ii) is decreasing in the number of parties N, and

(iii) converges to zero as N converges to infinity.

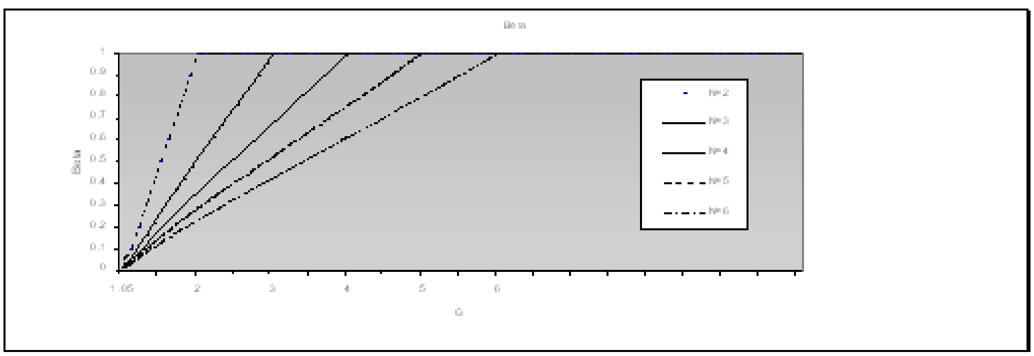

Figure 2: Probability of public good provision with $N = 2, ..., 6$ candidates.

The intuition for this result is that, as the number of parties increases, each party needs to focus his promises on a smaller and smaller set of voters. The public good thus becomes an increasingly disadvantageous way to compete in the electoral arena.

Figure 2 provides a numerical plot of the functions $\beta^* (G, N)$ as a function of G for $N = 2, ..., 6$.

4 Efficiency

4.1 Expected Welfare Maximization, Ex-Ante Efficiency, and Ex-Post Efficiency

We consider the perspective of a voter ex-ante. Before electoral promises are made, allocations are lotteries over consumption. Ex-ante Pareto efficiency ranks such allocations, taking the point of view of a voter who considers the expected utility of the outcome of the election before receiving an electoral promise. As the ex-ante perspective corresponds to evaluating alternatives behind a Rawlsian veil of ignorance, this perspective seems to be appropriate for thinking about issues of constitutional design.

Ex-ante, before receiving the promises, all voters face the same expected utility distribution in symmetric equilibria. Thus, in our equilibria, the ex-ante perspective coincides with the notion of expected surplus maximization. This is the notion that is conventionally used to evaluate the efficiency of public good provision (and gives rise to the "Samuelsonian condition" relating the marginal rate of transformation to the sum of the marginal rates of substitution).[19]

An alternative viewpoint is the ex-post one, which compares final allocations (after the uncertainty is resolved). After the winning policy is implemented, an allocation is a consumption vector that specifies how much each voter consumes. Ex-post Pareto efficiency ranks such allocations from the viewpoint of a voter who has already received this consumption. In our model, any redistributional allocation that gives transfers of more than G to at least one voter is ex-post Pareto optimal, as is the allocation in which the public good is provided. All equilibrium outcomes in our model, therefore, are ex-post Pareto optimal, even as the probability of providing the public good goes to zero (when the number of parties grows). This means that, in our context, ex-post Pareto efficiency cannot rank equilibrium allocations for different number of parties. This feature points to a more general property, which is that the outcome of electoral competition are always ex-post Pareto-efficient, as argued in Becker (1958). Because this efficiency property holds broadly across different institutional arrangements and size of the electorate,[20] the concept of Pareto efficiency is not commonly used in comparative

[19] In the popular textbook Stiglitz (1999), for example, pp. 169 and ff., the Samuelsonian criterion is used to evaluate the efficiency of the equilibrium in the median voter model. The explicit use of the Samuelsonian criterion to evaluate the efficiency of political equilibria goes back at least to Bowen (1943).

[20] With regards to the size of the electorate, consider a society in which only a small minority

politics exercises like ours. Despite its less frequent use, ex-post Pareto optimality remains a useful concept, and we do not argue against its use. In this paper, though, we adopt the conventional notion of welfare maximization since it yields a ranking that we think is intuitive.

4.2 Expected Welfare (or Ex Ante Efficiency) Ranking

In this section we rank the equilibrium outcomes of elections with N parties according to the expected welfare criterion. We consider a representative voter before he/she receives electoral promises, and ask whether he/she prefers a system with N parties to a system with $N+1$ parties. We show that all risk averse (or risk neutral) voters prefer elections with fewer parties.

If voters are risk neutral, they do not care about the (ex-ante) variance attached to a redistributive plan. In this case, efficiency is measured by the probability that the public good is provided in equilibrium. This probability has been shown to decrease with the number of parties. Therefore, when voters are risk neutral we already have a complete description of the (in)efficiency of increasing the number of parties. We now consider the welfare effects of the number of parties in the general case where voters may be risk averse. Assume that voters have a concave

(in the extreme, only one person) voted. The outcome of an election with only one voter would nevertheless be Pareto optimal for the entire society.

utility function defined over total consumption (transfers or public goods). In this case, voters care not only about the probability of public good provision, but also about the distribution of transfers at equilibrium. We show that any risk averse voter prefers a smaller number of parties.

When the public good is provided, denote the distribution of promises in consumption terms by $\mathbb{I}_{(-\infty,G)}(x)$ (the indicator function that is zero when $x < G$ and is one when $x \geq G$). Then, the expected c.d.f. of electoral promises by party i (in consumption terms) is

$$J_N(x) = \beta^*(G,N) \cdot \mathbb{I}_{(-\infty,G)}(x) + [1 - \beta^*(G,N)] \cdot F^*_{G,N}(x). \tag{1}$$

Since in equilibrium all parties have the same probability of winning, J_N is the expected distribution of electoral outcomes. We now show that all risk averse voters prefer J_N to J_{N+1}.

Proposition 1 *Consider any $N \geq 2$. For any G, all risk averse voters prefer an election with N parties to an election with $N+1$ parties.*

Proof: *Case* $G < 1$. In this case $\beta^*(G,N) = 0$, so $J_N(x) = F^*_{G,N}(x)$. If we denote $F^*_N(x) = (x/N)^{\frac{1}{N-1}}$, Theorem 1 yields $J_N(x) = F^*_N(x)$. We now show that $F^*_N(x)$ dominates $F^*_{N+1}(x)$ in the sense of second-order stochastic dominance. We do this by showing that there is a unique $\widetilde{x}_N \in (1,N)$ with the property that

$F_N^*(x) > F_{N+1}^*(x)$ if and only if $x > \widetilde{x}_N$. Write

$$
\begin{aligned}
F_N^*(x) - F_{N+1}^*(x) &= \left(\frac{x}{N}\right)^{\frac{1}{N-1}} - \left(\frac{x}{N+1}\right)^{\frac{1}{N}} \\
&= x^{\frac{1}{N}}\left[x^{\frac{1}{N(N-1)}}\left(\frac{1}{N}\right)^{\frac{1}{N-1}} - \left(\frac{1}{N+1}\right)^{\frac{1}{N}}\right]
\end{aligned}
$$

The expression in brackets is increasing in x, is negative for values of x close to zero and positive for $x = N$. Therefore, there exists a unique $\widetilde{x}_N$ at which $F_N^*(x) - F_{N+1}^*(x)$ equals zero. To verify that $\widetilde{x}_N > 1$, observe that when $x = 1$ the expression in brackets equals $(1/N)^{\frac{1}{N-1}} - (1/(N+1))^{\frac{1}{N}}$ which is negative because $(1/N)^{\frac{1}{N-1}}$ is an increasing function of N.

Case $1 < G < N + 1$. The proof of this case uses a similar technique to rank $J_N(x)$ and $J_{N+1}(x)$ in terms of stochastic dominance, but the argument is more involved and thus it is relegated to the appendix.

Case $G > N + 1$. In this case the public good is provided with probability 1 in both scenarios, so voters are indifferent. ∎

5 Institutional Features that Restrict the Extent of Electoral Competition

In our model it would be desirable to restrict the extent of electoral competition, for example by reducing the number of competing parties (ideally, to two). Here, we analyze—within our framework—some of the mechanisms, constitutional and otherwise, that restrict the extent of competition. The point of this section is not to suggest that all countries should adopt these institutional features. We merely wish to point out a possible efficiency rationale for some commonly observed institutions.

5.1 Barriers to Entry

Entry barriers can take the form of stipulations that a party must surpass a minimal vote share in order to be represented in a legislature (for instance, in Germany this threshold is 5%). Entry barriers can also be influenced by campaign finance laws, the ease of access to the media, as well as some intrinsic political forces that are not modeled in our paper.

To model the effect of barriers to entry, we now discuss a simple extension of our analysis to a world with endogenous entry of parties. In this world, the barrier to entry is literally interpreted as an entry cost. First, parties choose simultaneously and independently whether to enter or not. Then, the game of Section 2 is played among the entering parties. The payoff to a party who enters is the vote share minus the entry cost. The payoff to a party who does not enter is zero. In this game, there is an equilibrium with the property that the number of entering parties is decreasing in the cost of entry. Thus, higher entry costs can lead to a welfare improvement.

Our model displays excessive entry of parties. It is interesting to observe that the reason why entry is excessive is not—or not only—the duplication of fixed costs of entry. Rather, the inefficiency is due the equilibrium choice of electoral platforms. This is a crucial distinction with the analysis of entry in economic markets, where usually entry can have negative welfare effects because of the duplication of entry (or fixed) costs.[21] To push the point further, in economic markets typically a reduction in the cost of entry into a market leads to a welfare improvement. In particular, when entry costs converge to zero, we approach entry by an infinite number of firms, perfect competition, and a socially optimal allocation. In contrast, in our model, entry can be excessive *especially* if entry costs are low: the allocation when entry costs converge to zero is particularly inefficient because the probability of provision of the public good converges to zero.[22]

[21] See Tirole (1988). However, see Rosenthal (1980) for a market model in which more competitors result in a higher price even in the absence of fixed costs.

[22] An alternative model of entry that has the flavor of contestable markets is one in which an incumbent candidate chooses policy taking into account the possibility of entry. Having observed the policy, the entrant chooses whether to enter and what policy to offer. One could argue that when entry cost are zero, entry might force the incumbent to offer the efficient policy. Although this is true, it turns out that it does not translate into efficient outcomes: for values of the public good in the interesting range, the incumbent chooses the public good but the entrant enters, chooses to offer money, and obtains a majority. Thus, (1) entry is not deterred, (2) equilibrium is more inefficient than in the case where both candidates choose policies simultaneously.

5.2 Majoritarian Systems

Majoritarian systems, as defined by Lijphart (1984) and epitomized in the Westminster system, are political systems in which: (a) the electoral system is highly disproportional, in the sense that the distribution of seats in the assembly favors the party with a plurality of the votes and does not closely match the distribution of votes; (b) power is concentrated in one-party cabinets; and (c) there are two main parties. Exactly what features of majoritarian systems induce a small number of parties is debated: see Cox 1997. For our purposes, that is not crucial. We study an idealized majoritarian system in which the number of parties is taken as given (two), and parties compete in a winner-take-all election in which all the rewards, as well as the power to set policy, go to the party with a majority of the votes. The electoral incentives in this idealized majoritarian system differ from those discussed until now because we assume that, in the majoritarian system, parties maximize the probability of winning and not their vote share.

Let us compare the results in Section 3 to the equilibrium in a two-party election with a winner-take-all rule.[23] In our model, the comparison rests on the following

[23] In this, we follow Austen-Smith and Banks (1988), who compare the equilibrium in a proportional system with three parties with the equilibrium of a winner-take-all system with two parties. Austen-Smith and Banks analyze proportional representation in a three-candidate model of spatial competition that integrates the electoral and legislative processes.

theorem, proved in Lizzeri and Persico (2001).

Theorem 3 *Suppose $N = 2$. Under the winner-take-all system, in the unique equilibrium with equal treatment both parties offer the public good with probability $1/2$ for $G \in (1, 2)$. Parties offer the public good with probability zero if $G < 1$, and with probability one if $G > 2$.*

Comparing this result with the analysis in Section 3 shows that the proportional system is generally more inefficient than a winner-take-all system. The probability of public good provision is larger in the two-parties winner-take-all election relative to the equilibrium with N parties in the proportional system, unless $N = 2$ and $G < 3/2$, or $N = 3$ and G is between 1.963 and 2. This comparison suggests that two-party winner-take-all elections lead to more efficient outcomes than elections under the proportional system with more than 2 parties.[24]

[24] Of course, one must be careful to make a connection between this result and the extent of electoral competition. Majoritarian systems typically employ uninominal districts in which, it could be argued, electoral competition is fiercest. What is important is that fewer parties compete, and in this specific sense we say that the scope for electoral competition is more limited.

5.3 Run-off System

An electoral system that is used in France for local and national elections, and in a number of countries for local elections, is the run-off system.[25] In this system there are two rounds of voting. In the first round, voters choose among N parties. In the second round, voters choose among the two parties who got the highest vote shares in the second round. The run-off system does not, on its face, reduce the number of competing parties. In a strategic sense, however, the run-off system has some similarity to a two-party system. This is because in choosing whether to offer the public good or to offer money, parties must consider the fact that the second round involves competition with only one other party.

To model a run-off election we consider a simple modification of the model from Section 2. In this modification, parties commit in stage 1 to a platform (public good or distribution of money). Then voters vote in two rounds.[26] In the following result, we show that, for relatively high levels of G, a run-off system leads to outcomes that are more efficient than the system described in Section 3.

[25] See Osborne and Slivinsky (1996) for a model of the run-off system in a spatial context with citizen candidates.

[26] In many situations, an additional electoral rule augments the run-off electoral system described above. This is the provision that, if one candidate receives more than 50% of the votes in the first round, that candidate is elected without a second round. The statement of Theorem 4 is unchanged in the presence of the provision.

Theorem 4 *Suppose $G > 2$. Then, for any $N \geq 2$, under the run-off system there is an equilibrium where all parties offer to provide the public good with probability one.*

Proof: In the equilibrium, all parties win with probability $1/N$. Suppose party 1 deviates and offers money. Suppose he is among the two who pass to the second round. The other party who survives the first round offers the public good. Since $G > 2$, any distribution of money party 1 chooses will lead to a sure loss since the maximal share of the vote he can obtain is $1/G < 1/2$. Thus, offering money cannot pay. ∎

This result suggests that the number of parties in the first round overstates the effective degree of competition. Indeed, for $G > 2$ the probability of provision is the same as in the case of two parties and a single round of voting (Theorem 2). Thus, the run-off system favors the public good more than a system with only one round and mitigates the negative effects of the proliferation of parties.

The assumption that parties cannot change their platform after the first round of voting is not unrealistic, given the short time that separates the two rounds. The alternative assumption would be that the two first-round winners can freely renegotiate their platforms after the first round. In that case, the results of Theorem 4 are unchanged. Indeed, with second-round renegotiation, first-round promises become meaningless. So, irrespective of how voters vote and whoever passes to the second round, in the second round the two first-round winners will be competing in a winner-take-all election. In such an election, Theorem 3 guarantees that the equilibrium is efficient when $G > 2$, in line with the statement of Theorem 4.

6 Discussion of the Model

6.1 Targetability

Our model builds on the different characteristics of targetability of various projects. To make the point most clearly, in our model we put together two projects, one whose benefits and costs are perfectly targetable (redistribution) and one whose benefits and costs are completely non-targetable (the public good). This is of course a stark assumption. In reality, some fractions of the benefits or costs of the public good will frequently be targetable. It is possible to relax the assumption that the public good is completely non-targetable, by considering a model in which offering the public good does not require all the taxable resources of the economy, and thus the politician who promises the public good is left with some redistributive activity. For the case of two parties, this extension is presented in Lizzeri and Persico (1998). It can also be shown that the bounds for the value of the public good such that provision must be inefficient in equilibrium ($1 < G < N$) remain unchanged when the provision of the public good requires only part of the resources.

The ideas developed in this paper extend to richer environments in which there are several instruments with different level of tagetability. Suppose for instance that there are localities $l = 1, ..., L$, each comprising a fraction $1/L$ of the electorate. Suppose that, in addition to the general public good G, money can be used to purchase local public goods. Local public goods cost $1/L$ to produce, and give utility G_l to voters in locality L and zero to all other voters. Suppose $G_l < G$ for all l, which means that the most efficient option is to produce the general public good. In this case the equilibrium we have identified is unchanged, i.e., no candidate will want to deviate and offer G_l in any district. Indeed, offering G_l yields an expected payoff smaller than that of offering G in that locality, and the latter equals the expected payoff of offering transfers; this means that local public goods are dominated by transfers.

Suppose instead that $G_l > G$ for some l. A full analysis of this model is beyond the scope of this paper, but a benchmark result is easily obtained. When the number of parties N exceeds $\max_l \{G_l\}$, there is no equilibrium in which all parties promise the efficient allocation. Thus, when the number of parties gets large, the equilibrium features inefficient underprovision of (local and global) public goods, consistent with the main intuition of this paper.

6.2 Mixed Strategies

A notable feature of the equilibrium in our model is that parties employ mixed strategies.[27] Mixed strategy equilibria result from the presence of Condorcet cycles: any allocation of money can be defeated by another allocation which gives slightly more to most voters, at the expense of the small remaining fraction of voters (see the discussion in Section 3). In this sense, mixed strategy equilibria naturally arise in games of redistribution.

In our political setup, the presence of mixed strategies increases the level of abstraction of the model. In this section we discuss whether alternative modeling strategies can dispense with mixed strategy equilibria and still retain the main force of the paper, namely, the trade-off between targetability and efficiency.

One strategy is to resort to a purification argument.[28] Our game has a very large set of strategies and players, but one can apply a purification argument to a discretized version in which there is a finite number of voters V and transfers are constrained to be multiples of a small unit of account. It is convenient, moreover, to take the budget constraint to be satisfied in expectation instead of pointwise. With these assumptions, the strategies identified in Theorem 2 translate into an equilibrium for the discretized game for any value of V. It is then possible to obtain a

[27]Our model shares this feature with Myerson (1993), from which we borrow the model of redistribution.

[28]We are grateful to the editor for suggesting this strategy.

perturbed game (for given V) by adding small independent stochastic components to each candidate's payoff of each strategy. Each stochastic component represents the candidate's personal preference for a given allocation. These components are added to the candidate's "political" payoff from that allocation, and they are assumed to be the candidate's private information. In light of Harsanyi's purification theorem, the mixed strategy equilibrium of the discretized game can then be "purified," i.e., obtained as the limit of pure strategy equilibria of the perturbed game as the size of the stochastic components converges to zero.

The second strategy is to consider games with sequential moves. Our model posits that parties simultaneously commit to a platform. An alternative assumption would be to order parties, and have them commit sequentially, i.e., after observing the offers of the parties who precede them. Such a model is conducive to the existence of pure strategy equilibria. In the two-party game it is easy to find the (subgame-perfect) equilibrium. Suppose that $1 < G < 2$. The first mover will commit to the public good, knowing that if it chooses transfers it will receive (almost) no votes. The second mover will promise transfers of $G + \varepsilon$ to a fraction $1/(G - \varepsilon)$ of the electorate, and will win more than 50 % of the votes. This is the unique pure strategy equilibrium, and it is inefficient for the same reason as in the simultaneous-offer model we use. This intuition generalizes to the N party sequential-offer model, in which it remains true that when $1 < G < N$ it is not an equilibrium for all parties to promise the public good. In these dimensions, the sequential-offer model behaves much as the simultaneous in the sense that it gives similar intuitions. One drawback of the sequential-offer model is that it introduces artificial asymmetries among parties which influence their behavior; furthermore, a full analysis of the N-party case is fairly involved. Nonetheless, considering this sequential variant of our model allows us to make the point that while mixed strategy are a feature of the equilibrium in our simultaneous-offer model, the important economic forces highlighted by our model survive in a model where there is a pure strategy equilibrium. Thus, our results are not an artifact of mixed strategies.[29]

7 Conclusion

We have provided a tractable model of N-party electoral competition in which parties choose whether to offer to provide a public good or to target transfers to sub-groups of the population. Targetability is valuable to politicians who try to build a majority in an election, but it has no social value. Consequently, an inefficiency results where projects with diffuse benefits are underprovided by the political system because they are less targetable. We have shown that the incentive to offer particularistic platforms increases with the number of parties, and that,

[29] Dekel, Jackson, and Wolinsky (2004) discuss sequential vote buying games, and characterize pure strategy subgame-perfect equilibria of such games.

therefore, in equilibrium the probability that the public good is provided decreases with N. In fact, as N converges to infinity the public good is never provided, no matter how efficient it is. Voters standing behind a Rawlsian "veil of ignorance" prefer elections with smaller numbers of parties.

Our analysis can explain why it might be efficient to control the proliferation of parties in the electoral arena. Thus, the analysis has provided a potential efficiency rationale for electoral institutions that restrict the scope of electoral competition. We have suggested that institutions such as minimal vote share thresholds, the run-off electoral system, and majoritarian systems have desirable features because they act, in different ways, to limit the extent of electoral competition.

Several assumptions have been made to simplify the analysis. One is a particularly stylized treatment of the post-electoral coalitional bargaining. Another is that parties can perfectly commit to their platforms, and that they are motivated purely by electoral incentives. More technical, but still important, is that the public good is a discrete, all-or-nothing choice. We recognize that abstracting from these considerations makes the model more stylized, and possibly misses some interesting interactions. But these assumptions allowed us to present the intuition behind our results in the simplest manner. We believe that the main effect identified of our paper—specifically, the fact that the incentives to engage in special-interest politics tend to increase with the number of parties—would be robust to relaxing these assumptions.

Our analysis only attempts to describe one aspect of electoral competition. It has abstracted from ideological considerations in voting. This has been a useful simplification since it allowed us to present our results more clearly. We recognize, however, that in the presence of ideological cleavages there might be good reasons to have many parties, so that the political system can represent and give voice to diverse ideological constituencies. Also, we have barely mentioned corruption. Electoral competition may have the effect of reducing corruption.[30] A complete evaluation of the role of electoral competition should take into account the consequences of corruption as well as those discussed in the present paper.

[30] See Myerson (1993) for a model of multicandidate electoral competition that studies the performance of electoral systems in the presence of corrupt candidates.

Appendix

Proof of Theorem 2

(Part i). The probability of winning a vote with an offer of x in $[0,k]$ is

$$H^*(x) = [(1-\beta) F^*(x)]^{N-1}.$$

In view of the discussion in Section 3.1, in equilibrium it must be that $H^*(x) = x/N$ on $[0,k] \cup [G,N]$. Equating to x/N and solving for F^* yields

$$F^*_{G,N}(x) = \frac{1}{1-\beta}\left(\frac{x}{N}\right)^{\frac{1}{N-1}} \text{ for } x \in [0,k]. \tag{2}$$

The probability of winning a vote with an offer of x in $(G,N]$ is

$$H^*(x) = [(1-\beta) F^*(x) + \beta]^{N-1}.$$

Equating to x/N and solving for F^* yields

$$F^*_{G,N}(x) = \frac{1}{1-\beta}\left[\left(\frac{x}{N}\right)^{\frac{1}{N-1}} - \beta\right] \text{ for } x \in (G,N]. \tag{3}$$

To complete the characterization of $F^*_{G,N}$ we look for conditions to pin down β and k. The first condition is given by the continuity of $F^*_{G,N}$, which requires $F^*_{G,N}(k) = F^*_{G,N}(G)$. Substituting from (2) and (3), we get

$$\beta = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k}{N}\right)^{\frac{1}{N-1}}. \tag{4}$$

The second condition is given by the budget constraint, i.e.,

$$\int_0^k x dF^*_{G,N}(x) + \int_G^N x dF^*_{G,N}(x) = 1.$$

Substituting from (2) and (3), and computing the integral, we can express the budget constraint as

$$\frac{1}{1-\beta}\left[\left(\frac{k}{N}\right)^{\frac{N}{N-1}}+1-\left(\frac{G}{N}\right)^{\frac{N}{N-1}}\right]=1,$$

or, rearranging,

$$\beta=\left(\frac{G}{N}\right)^{\frac{N}{N-1}}-\left(\frac{k}{N}\right)^{\frac{N}{N-1}}. \tag{5}$$

Equations (4) and (5) form a system of two equations in the unknowns k and β. We denote the solution to this system by $k^*(G,N)$, $\beta^*(G,N)$.

We now show that the party behavior described in Theorem 2 indeed defines a Nash equilibrium. To this end, we show (a) that the redistributive strategy $F^*_{G,N}$ is a best redistributive strategy given the equilibrium behavior of other parties, and (b) that parties are indifferent between promising redistribution and public good.

To see that $F^*_{G,N}$ is an optimal redistributive strategy consider that, in view of the discussion in Section 3.1, in equilibrium $H^*(x)=x/N$ on $[0,k^*(G,N)]\cup[G,N]$. When H^* has this form, the payoff of a candidate who redistributes according to F is

$$\begin{aligned}&\int_0^\infty H^*(x)\,dF(x)\\ \leq\ &\int_0^\infty \frac{x}{N}dF(x)=\frac{1}{N},\end{aligned}$$

and the strict inequality holds only if F has a larger support than H^*. In particular, $F^*_{G,N}$ attains the maximal possible payoff of $1/N$. Let us now compute the payoff from promising the public good. The expected vote share from offering the public good, S_G, is equal to

$$S_G = \sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} \left[(1-\beta) F^* (G) \right]^{N-1-j} \beta^j.$$

Using the fact that

$$\sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} q^{N-1-j} p^j = \left[(q+p)^N - q^N \right] / pN,$$

we rewrite the above expression as

$$\frac{\left[(1-\beta) F^* (G) + \beta \right]^N - \left[(1-\beta) F^* (G) \right]^N}{\beta N}. \tag{6}$$

Since $F^* (G) = F^* (k)$ we can rewrite this as

$$\frac{\left[(1-\beta) F^* (G) + \beta \right]^N - \left[(1-\beta) F^* (k) \right]^N}{\beta N},$$

and after substituting from (2) and (3), we obtain

$$\frac{\left[\frac{G}{N} \right]^{\frac{N}{N-1}} - \left[\frac{k}{N} \right]^{\frac{N}{N-1}}}{\beta N}.$$

When $\beta = \beta^* (G, N), k = k^* (G, N)$, this expression equals $1/N$ in light of equation (5). Thus, it is indeed the case that in equilibrium $S_G = 1/N$. This concludes the (constructive) proof of existence of equilibrium.

Uniqueness follows from the analysis in Section 3.1 together with the uniqueness of the solutions of β and k.

We now to a characterization of $k^*(G,N)$. From (4) and (5), $k^*(G,N)$ must solve

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right)=\left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right). \tag{7}$$

The function $h(z)=(z/N)^{\frac{1}{N-1}}(1-(z/N))$ is single-peaked on $[0,\infty)$, has a maximum at $z=1$, and has value zero at $z=0$ and $z=N$. Because h is single peaked, equation (7) only has two solutions: one is $k=G$; the other is $k^*(G,N)=h^{-1}(G)$, where h^{-1} denote the inverse of h on the interval $[0,1]$. Only the second solution can be part of an equilibrium, since the first solution requires that β equals zero, and that is impossible when $1<G<N$. Solving for $\beta^*(G,N)$ is accomplished by substituting $k^*(G,N)$ into equation (4).

Observe that $k^*(G,N)$ is decreasing in G since $h(z)$ is increasing on $(0,1)$ and decreasing on $(1,N)$. Therefore, $\beta^*(G,N)$ is increasing in G.

(Part ii). Recalling equations (5) and (7), we have that, for $0<k<1<G<N$

$$\beta = \left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k}{N}\right)^{\frac{1}{N-1}} \tag{8}$$

$$\left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right) = \left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right) \tag{9}$$

This proof of this part now proceed through a series of intermediate steps

Lemma 2 *Suppose that* $0<k<1<G<N$ *and that* k *is given by equation (9), then* $k+G\geq 2$.

Proof: We can rewrite equation 7 as $k\left(N-k\right)^{N-1}=G\left(N-G\right)^{N-1}$. We show that the program $\min_{k,G}\left(k+G-2\right)$ subject to the constraint $k\left(N-k\right)^{N-1}=G\left(N-G\right)^{N-1}$ has a non negative value. The Lagrangean and the associated first order conditions for this problem are:

$$\begin{aligned}
\mathcal{L} &= \left(k+G-2\right)+\lambda\left[k\left(N-k\right)^{N-1}-G\left(N-G\right)^{N-1}\right] \\
\frac{\partial\mathcal{L}}{\partial k} &= 1+\lambda\left[\left(N-k\right)^{N-1}-k\left(N-1\right)\left(N-k\right)^{N-2}\right]=0 \\
\frac{\partial\mathcal{L}}{\partial G} &= 1-\lambda\left[\left(N-G\right)^{N-1}-G\left(N-1\right)\left(N-G\right)^{N-2}\right]=0
\end{aligned}$$

The first order conditions must be satisfied if there is an interior solution $0<k<1<G<N$. Manipulating the first order conditions we obtain

$$\begin{aligned}
\left(N-k\right)^{N-1}-k\left(N-1\right)\left(N-k\right)^{N-2} &= -\left(N-G\right)^{N-1}+G\left(N-1\right)\left(N-G\right)^{N-2} \\
\frac{\left(1-k\right)}{k}\left(\frac{1}{N-k}\right) &= -\frac{\left(1-G\right)}{G}\left(\frac{1}{N-G}\right)
\end{aligned}$$

Define the function

$$f\left(x\right)=\frac{\left(1-x\right)}{x}\left(\frac{1}{N-x}\right).$$

A necessary condition for an interior maximum is that $f\left(k\right)+f\left(G\right)=0$. So, any interior pair k,G that solves the original problem also solves the following auxiliary problem

$$\min_{k,G}\left(k+G-2\right)\ \text{ s.t. } f\left(k\right)+f\left(G\right)=0.$$

The value of this problem is not higher (and possibly, lower) than the original problem. The Lagrangean for this new problem are

$$\widetilde{\mathcal{L}} = (k + G - 2) + \lambda \left[\frac{(1-k)}{k} \left(\frac{1}{N-k} \right) + \frac{(1-G)}{G} \left(\frac{1}{N-G} \right) \right].$$

The first order conditions are

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial k} = 1 + \lambda \left[\frac{-(k-1)^2 - N + 1}{(N-k)^2 k^2} \right] = 0,$$

and similarly

$$\frac{\partial \widetilde{\mathcal{L}}}{\partial G} = 1 + \lambda \left[\frac{-(G-1)^2 - N + 1}{(N-G)^2 G^2} \right] = 0.$$

Combining the first order conditions yields

$$\frac{-(k-1)^2 - N + 1}{(N-k)^2 k^2} = \frac{-(G-1)^2 - N + 1}{(N-G)^2 G^2}. \tag{10}$$

Now, from $f(k) = -f(G)$ we have

$$G(N-G)(1-k) = -k(N-k)(1-G),$$

and squaring

$$G^2 (N-G)^2 (1-k)^2 = k^2 (N-k)^2 (1-G)^2.$$

Substituting into (10) yields

$$\frac{\left(N - 1 + (k-1)^2\right)}{(k-1)^2} = \frac{\left(N - 1 + (G-1)^2\right)}{(G-1)^2}. \tag{11}$$

The function $b(z) = (N - 1 + z)/z$ is strictly decreasing in z (at least for $z \geq 0$) so the only way to satisfy equation (11) is to have $(k-1)^2 = (G-1)^2$, or, equivalently (since $k \neq G$), $1-k = G-1$. This equation yields $k+G=2$, which shows that the value of the auxiliary minimization problem cannot be smaller than zero at an interior minimum. It is immediate to verify that boundary solutions are not a problem, so we get that the value of the auxiliary minimization problem cannot go below zero, and therefore the original minimization problem also cannot go below zero. This concludes the proof.

Proposition 2 $k^*(G,N)$ *is increasing in* N

Proof: (9) is equivalent to

$$\log(k) + (N-1)\log(N-k) = \log(G) + (N-1)\log(N-G)$$

Totally differentiating w.r.to N and rearranging yields

$$\left(\frac{N(1-k)}{(N-k)k}\right)\frac{\partial k}{\partial N} = \ln(N-G) - \ln(N-k) + \frac{(N-1)(G-k)}{(N-G)(N-k)}.$$

The sign of $\partial k/\partial N$ equals the sign of the RHS. Notice that, for fixed k and G, the RHS converges to zero as $N \to \infty$. So, if we are able to show that the RHS is decreasing as a function of N for fixed k and G then we have proved that the RHS is positive for finite N, which concludes the proof. The derivative with respect to N of the RHS equals

$$\frac{G-k}{(N-G)^2(N-k)^2}\left[-N(G+k-2)+k(G-1)+G(k-1)\right]$$

and the sign of the above expression is the sign of the expression in brackets. Now, since by the above Lemma $G+k-2\geq 0$, then

$$\begin{aligned} -N\left(G+k-2\right)+k\left(G-1\right)+G\left(k-1\right) &\leq -\left(G+k-2\right)+k\left(G-1\right)+G\left(k-1\right) \\ &= 2\left(G-1\right)\left(k-1\right)\leq 0 \end{aligned}$$

and inequality holds strictly when $k,G\neq 1$.

Substituting equation (9) into equation (8) we get

$$\beta=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(\frac{G-k}{N-k}\right).$$

To show that $d\beta/dN\leq 0$ it suffices to show that $d\log\left(\beta\right)/dN\leq 0$.

$$\frac{d\log\left(\beta\right)}{dN}=\frac{\ln\left(N\right)-\ln\left(G\right)}{\left(N-1\right)^{2}}-\frac{1}{\left(N-1\right)N}-\frac{1}{N-k}-\left(\frac{N-G}{\left(G-k\right)\left(N-k\right)}\right)\frac{\partial k}{\partial N}.$$

Since $\partial k/\partial N\geq 0$,

$$\begin{aligned} \frac{d\log\left(\beta\right)}{dN} &\leq \frac{\ln\left(N\right)-\ln\left(G\right)}{\left(N-1\right)^{2}}-\frac{1}{\left(N-1\right)N}-\frac{1}{N-k} \\ &\leq \frac{\ln\left(N\right)}{\left(N-1\right)^{2}}-\frac{1}{\left(N-1\right)N}-\frac{1}{N}. \end{aligned}$$

Multiplying both sides by $N\left(N-1\right)^{2}$ yields

$$\begin{aligned} N\left(N-1\right)^{2}\left(\frac{d\log\left(\beta\right)}{dN}\right) &\leq N\ln\left(N\right)-\left(N-1\right)-\left(N-1\right)^{2} \\ &= N\left[\ln\left(N\right)-\left(N-1\right)\right]\leq 0 \end{aligned}$$

where the inequality follows because, for all $x>0$, it is the case that $\ln\left(x\right)\leq x-1$. This shows that $d\log\left(\beta\right)/dN\leq 0$.

(Part iii). First, recall from that $k^*(G,N) \in (0,1)$, and therefore $(1-(k^*(G,N)/N)) \to 1$. Since $k^*(G,N)$ solves

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right)=\left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right)$$

and the left hand side converges to 1 as $N \to \infty$, it must be that $(k^*(G,N)/N)^{\frac{1}{N-1}} \to 1$. Noticing that

$$\beta^*(G,N)=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k^*(G,N)}{N}\right)^{\frac{1}{N-1}}$$

and $(G/N)^{\frac{1}{N-1}} \to 1$ yields the conclusion.

Proof of Proposition 1

Substituting into (1) from (2) and (3) yields

$$J_N(x)=\begin{cases}\left(\frac{x}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [0,k^*(G,N)) \\ \left(\frac{k^*(G,N)}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [k^*(G,N),G) \\ \left(\frac{x}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [G,N]\end{cases}$$

Observe that $J_N(x) = F_N^*(x)$ on $[0,k^*(G,N)) \cup [G,N]$. We show that $J_N(x)$ dominates $J_{N+1}(x)$ in the sense of second-order stochastic monotonic dominance (see Huang and Litzenberger (1988)). This requires proving that: (i) the expected consumption (transfers or public good) is no smaller under $J_N(x)$ than under $J_{N+1}(x)$; and (ii) that $S(z) := \int_0^z [J_N(x) - J_{N+1}(x)]\,dx \leq 0$ for all $z \geq 0$. Part (i) is straightforward given the fact that $\beta^*(G,N)$ is decreasing in N. We prove part (ii) by showing that $J_N(x)$ crosses $J_{N+1}(x)$ only once, and from below, for $x \in (0, N+1)$. We divide the proof into two subcases.

1. $G < \widetilde{x}_N$.

   In this case, in view of Proposition 1 part (i), $J_N(x)$ crosses $J_{N+1}(x)$ exactly once, and from below, on $x \in (G, N+1)$. Therefore, we must check that, for $x \in (0, G)$ $J_N(x)$ is smaller than $J_{N+1}(x)$. For any $n \geq 2$, the function $J_n(x)$ is constant and equal to $F_n^*(G) - \beta^*(G, n)$ on $(k^*(G, n), G)$. Since by assumption $G < \widetilde{x}_N$, we have $F_N^*(G) < F_{N+1}^*(G)$. Recalling that $\beta^*(G, N) > \beta^*(G, N+1)$, we conclude that $J_N(x) < J_{N+1}(x)$ for values of x between $\max\{k^*(G, N), k^*(G, N+1)\}$ and G.

   For values of x smaller than $\max\{k^*(G, N), k^*(G, N+1)\}$ it is easily verified that $J_N(x) < J_{N+1}(x)$ because $F_N^*(x) < F_{N+1}^*(x)$ (see Figure 3).

2. $G > \widetilde{x}_N$.

In this case, in view of Lemma 1, $J_N(x)$ is always greater than $J_{N+1}(x)$ on $x \in (G, N+1)$. So, it suffices to check that $J_N(x)$ crosses $J_{N+1}(x)$ at most once, and then from below, on $(0, G)$. To this end, observe that if $J_N(x)$ crosses $J_{N+1}(x)$ on $(0, G)$, any crossing must happen on $(0, 1)$ because for all n, $J_n(x)$ is constant on $(k^*(G, n), G)$ and $k^*(G, n) < 1$. Now, take the smallest point $\widehat{x}$ in $(0, 1)$ at which

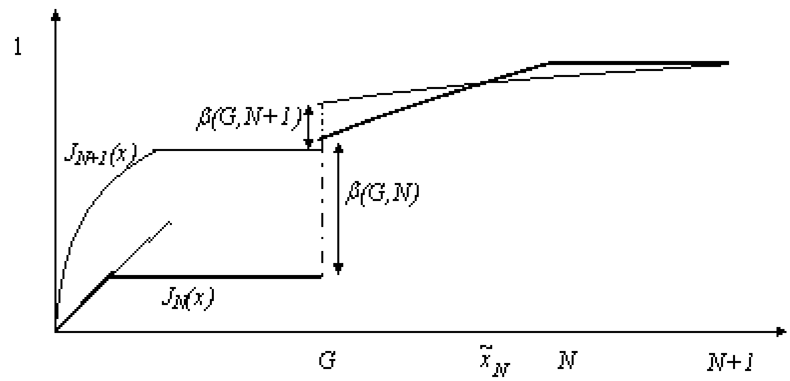

Figure 3: Case $G < \widetilde{x}_N$.

$J_N(x)$ crosses $J_{N+1}(x)$. On $(0,1)$, we have $F^*_{N+1}(x) > F^*_N(x)$ (indeed, $\widetilde{x}_n > 1$ for all n by Lemma 1). Therefore, $\widehat{x}$ must be such that $J_{N+1}(\widehat{x}) < F^*_{N+1}(\widehat{x})$, that is, $J_{N+1}(x)$ is constant for x between $\widehat{x}$ and G. But then there can be no further crossings between $J_N(x)$ and $J_{N+1}(x)$ on $(\widehat{x}, G)$, which proves uniqueness of the crossing. To show that the crossing is from below, notice that $\widehat{x}$ is, by construction, the smallest point at which the two functions cross, and, for x small enough, $J_{N+1}(x) = F^*_{N+1}(x) > F^*_N(x) = J_N(x)$.

References

Alesina, Alberto, Sule Ozler, Nouriel Roubini, and Philip Swagel (1996). "Political instability and economic growth." *Journal of Economic Growth* 1, pp. 189-211.

Austen-Smith, David and Banks, Jeffrey (1988) . "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review*, Vol. 82, pp. 405-422.

Baron, David (1991) "Majoritarian Incentives, Pork Barrel Programs and Procedural Control." *American Journal of Political Science* p. 57-90.

Bates, Robert H.(1981) *Markets and States in Tropical Africa.* University of California Press, .

Becker, Gary S. (1958) "Competition and Democracy." *Journal of Law and Economics* 1: pp. 105-109.

Besley, Timothy and Stephen Coate, (1997) "An Economic Model of Representative Democracy" *Quarterly Journal of Economics* 112(1), p. 85-114.

Besley, Timothy and Coate, Stephen (1998), "Sources of Inefficiency in a Representative Democracy: a Dynamic Analysis." *American Economic Review, 88*(1), pp. 139-56.

Bowen, Howard (1943) "The interpretation of voting in the allocation of economic resources" *Quarterly Journal of Economics* p. 27-48.

Buchanan, James M. (1968) "Democracy and Duopoly: A Comparison of Analytical Models." *The American Economic Review 58*(2) pp. 322-31.

Buchanan, James and Gordon Tullock (1962) *The calculus of consent* University of Michigan Press.

Campos, Nauro F. and Jeffrey B. Nugent (2002), . "Who is afraid of political instability?" *Journal of Development Economics*, *67*(1) pp. 157-172.

Chari, V.V. Jones, Larry and Marimon, Ramon (1997) "The Economics of Split-ticket voting in Representative Democracies," *The American Economic Review* Vol. 87 No. 5.

Coate, Stephen and Morris, Stephen (1995) "On the Form of Transfers to Special Interests." *Journal of Political Economy, 103*(6), pp.1210-35.

Cox, Gary W. (1987) "Electoral Equilibrium Under Alternative Voting Institutions." *American Journal of Political Science* Vol. 31, pp. 82-108.

Cox, Gary W. (1987) *The Efficient Secret.* Cambridge: Cambridge University Press.

Cox, Gary W. (1997) *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge: Cambridge University Press.

Cox, Gary W (2001). "Public Goods, Targetable Goods, and Electoral Competition." Preliminary Draft, UC San Diego.

Coughlin, Peter (1992) *Probabilistic Voting Theory.* Cambridge: Cambridge University Press.

Dekel, Eddie, Matthew Jackson, and Asher Wolinsky (2004) "Vote Buying." Manuscript Caltech.

Diermeier, Daniel and Antonio Merlo (2000), "Government Turnover in Parliamentary Democracies," *Journal of Economic Theory*, vol. 49, pp. 46-79.

Diermeier, Daniel, Hulya Eraslan, and Antonio Merlo. (2002) "Coalition Governments and Comparative Constitutional Design." *European Economic Review* 46, pp. 893-907

Diermeier, Daniel, Hulya Eraslan, and Antonio Merlo (2003) "A Structural Model of Government Formation." *Econometrica* 71, pp. 27-70

Duverger, M. (1954) *Political Parties: Their Organization and Activity in the Modern State.* Wiley, New York.

Feddersen, Timothy J; Pesendorfer, Wolfgang. (1996) "The Swing Voter's Curse." *American Economic Review.* Vol. 86 (3). p 408-24.

Feddersen, Timothy; Pesendorfer, Wolfgang (1997). "Voting Behavior and Information Aggregation in Elections with Private Information." *Econometrica.* Vol. 65 (5). p 1029-58.

Huang, Chi-fu and Robert Litzenberger (1988). *Foundations for Financial Economics.* Prentice-Hall, Englewood Cliffs, New Jersey.

Lijphart Arend (1984) *Democracies: Patterns of Majoritarian and Consensus Government in Twenty-One countries*, New Haven: Yale University Press.

Lindbeck, Assar and Weibull, Jurgen (1987). "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice, 52,* pp. 273-297.

Lipset, Seymour and Stein Rokkan (1967) "Cleavage Structures, Party Systems, and Voter Alignments: An Introduction," in Lipset and Rokkan, eds. *Party Systems and Voter Alignments*, Free Press, New York, pp. 1-64.

Lizzeri, Alessandro and Persico, Nicola (1998). "The Provision of Public Goods Under Alternative Electoral Incentives." CARESS Working Paper # 98-08, University of Pennslvania.

Lizzeri, Alessandro and Persico, Nicola (2001). "The Provision of Public Goods Under Alternative Electoral Incentives." *The American Economic Review.*

Milesi-Ferretti, Gian Maria; Perotti, Roberto, and Rostagno, Massimo (2000). "Electoral Systems and the Composition of Public Spending." Columbia University, mimeo.

Myerson, Roger. B. (1993) "Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems", *American Political Science Review* Vol. 87, No. 4.

Myerson, Roger B. (1993) "Effectiveness of Electoral Systems for Reducing Government Corruption: A Game-Theoretic Analysis." *Games & Economic Behavior 5* (1) p 118-32.

Ordeshook, Peter C., and Olga V. Shvetsova. (1994) "Ethnic Heterogeneity, District Magnitude, and the Number of Parties." *American Journal of Political Science*, *38*(1), pp. 100-23.

Osborne, Martin and Al Slivinski, (1996) "A Model of Political Competition with Citizen-parties" *Quarterly Journal of Economics 111*(1) p. 65-96.

Palfrey, Thomas R. (1984) "Spatial Equilibrium with Entry." *The Review of Economic Studies*, *51*(1), pp. 139-156.

Persson, Torsten and Tabellini, Guido.(1999) Alfred Marshall Lecture "The Size and Scope of Government: Comparative Politics with Rational Politicians." *European Economic Review* , *43*(4-6), pp. 699-735.

———————— (2000a) "Political Institutions and Policy Outcomes: What Are the Stylized Facts?" Mimeo, IIES .

————————. 2000(b). *Political Economics-Explaining Economic Policy.* MIT Press, Cambridge, MA,.

Persson, Torsten, Roland, Gerard, and Tabellini, Guido (2000) "Comparative Politics and Public Finance." *Journal of Political Economy 108*(6) pp. 1121-1161.

Rosenthal, Robert W.(1980) "A Model in which an Increase in the Number of Sellers Leads to a Higher Price." *Econometrica*, *48*(6) pp. 1575-1580.

Shepsle, Kenneth A. (1991) *Models of Multiparty Electoral Competition* (Fundamentals of Pure and Applied Economics. Political Science and Economics Section, Vol 45) Harwood Academic Publishers.

Stiglitz, Joseph E. (1999) *Economics of the Public Sector.* New York: Norton (2nd ed.).

Usher, Dan (1994) "The Significance of the Probabilistic Voting Theorem" *The Canadian Journal of Economics*, 27, 2 pp. 433-445.

Weingast, Barry, Shepsle, Kenneth, and Johnsen, Cristopher (1981) "The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics." *Journal of Political Economy*, *89*, pp. 642-64.